WAGNER'S BEAM CYCLE

N.S. TRAHAIR

RESEARCH REPORT R916
MARCH 2011

ISSN 1833-2781

SCHOOL OF CIVIL ENGINEERING

SCHOOL OF CIVIL ENGINEERING

WAGNER'S BEAM CYCLE

RESEARCH REPORT R916

N S TRAHAIR

MARCH 2011

ISSN 1833-2781

School of Civil Engineering, Research Report R916
Wagner's Beam Cycle
N S Trahair BSc BE MEngSc PhD DEng
March 2011

ISSN 1833-2781

Published by:
School of Civil Engineering
The University of Sydney
Sydney NSW 2006
Australia

This report and other Research Reports published by the School of Civil Engineering are available at http://sydney.edu.au/civil

ABSTRACT

This paper summarises a number of research studies on the torsion and buckling behaviour of beams which derive from a theory developed by Wagner, who extended Timoshenko's treatment of the elastic buckling of I-section beams and columns to members of general thin-walled open cross-section. These studies include applications of the first-order Wagner theory to the buckling of beams and cantilevers, and of the second-order Wagner theory to the large rotations and post-buckling behaviour of beams.

KEYWORDS

Beams, bending, buckling, large rotations, post-buckling, steel, thin-walled sections, torsion.

TABLE OF CONTENTS

ABSTRACT 3
KEYWORDS 3
TABLE OF CONTENTS 4
1 INTRODUCTION 5
2 BUCKLING OF MONOSYMMETRIC MEMBERS 5
2.1 Beams 5
2.2 Cantilevers 6
2.3 Inelastic Beams 6
2.4 Arches 6
3 SECOND-ORDER WAGNER EFFECTS 6
3.1 Inelastic Torsion 7
3.2 Post-Buckling Of Beams 7
3.3 Beam Design Curves 7
4 CONCLUSIONS 7
APPENDIX 1 - REFERENCES 8
APPENDIX II - NOTATION 9
FIGURES 10

1 INTRODUCTION

Wagner (1) is generally credited with extending Timoshenko's (2) treatment of the elastic buckling of I-section beams and columns to members of general thin-walled open cross-section.

A feature of Wagner's treatment is the prediction of disturbing torques which lead for example to the torsional buckling of cruciform columns, as shown in Fig. 1. These torques arise from transverse components of the axial stresses in the twisted longitudinal fibres of a member which act about the shear centre axis, as shown in Fig. 2. When the stresses are compressive, the torque increases the twisting, and reduces the effective resistance to uniform torsion from $GJ\phi'$ to $(GJ\phi' - Pr_0^2\phi')$, in which G is the shear modulus of elasticity, J is the uniform torsion section constant, ϕ' is the twist rotation per unit length, P is the compression load, and r_0 is the polar radius of gyration $r_0 = \sqrt{((I_x+I_y)/A)}$, in which I_x and I_y are the principal axis second moments of area and A is the area of the section. This resistance reduces to zero and the column buckles torsionally when $P = GJ/r_0^2$.

2 BUCKLING OF MONOSYMMETRIC MEMBERS

2.1 BEAMS

The application of Wagner's treatment to the lateral buckling of simply supported monosymmetric I-beams in uniform bending leads to the prediction of the elastic buckling moment M as satisfying

$$\frac{M}{M_{yz}} = \sqrt{1 + \left(\frac{\beta_x P_y}{2M_{yz}}\right)^2} + \left(\frac{\beta_x P_y}{2M_{yz}}\right) \tag{1}$$

in which M_{yz} is given by

$$M_{yz} = \sqrt{\left(\frac{\pi^2 EI_y}{L^2}\right)\left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \tag{2}$$

in which E is the Young's modulus of elasticity, L is the length, I_w is the warping section constant, and β_x is the monosymmetry section constant given by

$$\beta_x = \frac{\int y(x^2 + y^2)\mathrm{d}A}{I_x} - 2y_0 \tag{3}$$

in which y_0 is the shear centre coordinate.

For a beam with equal flanges, $\beta_x = 0$, and the disturbing torque caused by the compression flange stresses is balanced by the restoring torque caused by the tension flange stresses, so that the elastic buckling moment is equal to M_{yz}. For a beam whose compression flange is the larger, the tension stresses in the smaller flange dominate the monosymmetry effect because not only do the tension flange fibres rotate further during twisting, their forces also have greater lever arms about the shear centre axis, as shown in Fig. 3. In this case, β_x is positive, and $M > M_{yz}$. The converse is true for a beam whose compression flange is the smaller.

These effects of monosymmetry agree qualitatively with the simple concept of relating the beam buckling moment directly to the flexural buckling of the compression flange as a column. Thus it is advantageous to use more material in the compression flange to increase its column buckling resistance. This conclusion is reinforced by the fact that the compression flange buckles the further, as shown in Fig. 4, so that increasing its stiffness increases the beam buckling resistance.

Not all writers have agreed with this treatment, with Bleich (3) of the opinion that the buckling of monosymmetric beams could be predicted by using the predictions for doubly symmetric beams, which is equivalent to assuming $\beta_x = 0$ so that M_{yz} becomes the predicted buckling moment.

2.2 CANTILEVERS

Cantilevers differ somewhat from simply supported beams, in that it is the tension flange which buckles the further, as shown in Fig. 5 (4). Further, uniform bending of cantilevers rarely occurs, if ever, and the critical practical loading is that of a concentrated end load, which introduces the effect of load height, in which the buckling resistance decreases as the load height above the shear centre increases.

The effects of monosymmetry (and of load height) on the buckling of beams and cantilevers were investigated analytically and experimentally by Anderson (5). His correlations between analysis and experiment for cantilevers shown in Fig. 6 provide convincing evidence for the Wagner effect.

2.3 INELASTIC BEAMS

The Wagner effect influences the inelastic buckling of a steel beam, in that the combination of the anti-symmetric bending strains with symmetric residual strains causes different yield patterns in the flanges, so that the remaining elastic regions are monosymmetric. When the bending moment distribution varies along the beam, the elastic regions are tapered as well as monosymmetric, as shown in Fig. 7 (6). As a preliminary to his investigations of the inelastic buckling of steel beams, Kitipornchai (7) analysed and tested the elastic buckling of tapered monosymmetric beams, as shown in Fig. 8, again providing convincing evidence for the Wagner effect.

2.4 ARCHES

The Wagner effect on the flexural-torsional buckling of monosymmetric arches under point loads was studied analytically and experimentally by Papangelis (8). His results shown in Fig. 9 also provide convincing evidence for the Wagner effect, as well as for his analytical predictions.

3 SECOND-ORDER WAGNER EFFECTS

The Wagner effects described above influence the stability of columns and beams. They are torque effects that are proportional to the product of the twists ϕ' and the loads P or moments M, and might be described as first-order Wagner effects. There are other Wagner effects present during large twists, even when there are no loads or moments (9). These might be referred to as second-order Wagner effects.

For members under pure torsion, the second-order Wagner effect is given by the third term on the right-hand side of the torsion equation (10)

$$M_z = GJ\phi' - EI_w\phi''' + \frac{1}{2}EI_n(\phi')^3 \tag{4}$$

in which ' indicates differentiation of the twist rotation ϕ with respect to the distance z along the member, and I_n is the "Wagner" section constant (9). For doubly symmetric I-sections, I_n is given by

$$I_n = \int (x^2 + y^2)^2 \,\mathrm{d}A - \frac{\{\int (x^2 + y^2)\mathrm{d}A\}^2}{A} \tag{5}$$

This third term represents the torque effect of an internal stress resultant which has been called a "Wagner". It provides a stiffening effect which becomes appreciable at large twist rotations, as shown in Fig. 10.

The origin of the "Wagner" is demonstrated in Fig. 11 by the axial shortening of the twisted fibres of a thin rectangular section cantilever. Each fibre becomes a helix whose projected length on the z axis shortens as the twist increases. If unrestrained, these fibre shortenings would vary across the end section, as indicated, producing gross shear straining. This shear straining is prevented by axial tensile stresses which increase the developed length of the fibres further from the axis of twist and by compressive stresses which decrease the developed length of the fibres closer to the axis of twist. The axial resultant of these stresses must be zero because there is no external force acting, but the set of stresses make a non-zero Wagner contribution to the total torque resistance (positive because the tensile stresses further from the axis of twist make the dominant contribution).

3.1 INELASTIC TORSION

Physical evidence of the second-order Wagner effect was provided by tests by Farwell (11) on simply supported steel I-beams with symmetrical torsion loads (Fig. 12). At moderate torques, yielding causes the twist rotations to increase significantly, but at higher torques, the beams stiffen, as shown in Fig. 13. Final failure of the beams was due to tensile fracture at the flange tips, at torques considerably higher than upper bounds to those that cause plastic collapse (12).

3.2 POST-BUCKLING OF BEAMS

It is the second-order Wagner effect that at least partially ensures that the post-buckling behaviour of beams and cantilevers is imperfection insensitive, as shown by the slowly rising post-buckling curves of Fig. 14 (4).

The post-buckling of redundant beams was investigated first by Masur and Milbradt (13), who showed that there was a significant and favourable redistribution of the moments in narrow rectangular beams as the twist rotations increased, as shown in Fig. 15. Subsequent investigations by Woolcock (14, 15) indicated that the redistributions in practical I-section beams take place too slowly to lead to significant strength increases.

3.3 BEAM DESIGN CURVES

Despite the finding that post-buckling redistributions are slow in practical I-beams, it is worth considering what may happen to a beam under gross twist rotations. When the beam supports gravity loading, the worst that can happen is that the maximum moment section rotates through 90^o in which case the moment acts about the minor axis, as shown in Fig. 16. Thus the minimum strength of a slender beam bent about its major axis is its minor axis strength, which may be significantly higher than its predicted elastic buckling moment, as shown in Fig. 17 (16). In this case, the elastic buckling load has a serviceability significance, in that it suggests a load at which deflections become excessive.

A similar conclusion can be reached for angle lintels, for which there is the added complication that the applied loads cause primary torsion (17). In the case of lintels with the horizontal leg down, twist rotations initially strengthen the lintel by causing its stiffer principal plane to rotate towards the plane of the loads, as shown in Fig. 18.

In equal angle lintels with the horizontal leg up, twist rotations of 45^o cause the applied loading to cause bending about the minor axis, as shown in Fig. 19, for which the lintel strength is 85% of the strength of a fully restrained lintel. This minor axis strength may be significantly higher than the current design strength based on the load at which large rotations occur.

4 CONCLUSIONS

This paper summarises a number of research studies on the torsion and buckling behaviour of beams which derive from a theory developed by Wagner, who extended Timoshenko's treatment of the elastic buckling of I-section beams and columns to members of general thin-walled open cross-section.

The first-order Wagner effect leads to the torsional buckling of cruciform columns, and modifies the flexural-torsional buckling of monosymmetric beams, cantilevers, and arches. Theoretical predictions have been confirmed by test results.

The second-order Wagner effect becomes important at large twist rotations. While large twist rotations do not occur in well-designed structures, the existence of the second-order Wagner effect shows that the post-buckling of beams is imperfection insensitive, suggests that the design strengths of very slender beams are equal to their minor axis strengths, and provides assurance that approximate plastic collapse analyses of torsion will be conservative.

APPENDIX 1 - REFERENCES

(1) Wagner, H, 'Verdrehung und Knickung von Offenen Profilen (Torsion and Buckling of Open Sections)', *25th Anniversary Publication*, Technische Hochschule, Danzig, 1904-1929, Translated as Technical Memorandum No. 87, National Committee for Aeronautics, 1936.

(2) Timoshenko, SP, 'Einige Stabilitaetsprobleme der Elastizitaetstheorie', *Collected Papers of Stephen P Timoshenko*, McGraw-Hill, New York, 1953, pp 1-50.

(3) Bleich, F, *Buckling Strength of Metal Structures*, McGraw-Hill, New York, 1952.

(4) Woolcock, ST and Trahair, NS, 'Post-Buckling Behaviour of Determinate Beams', *Journ. Eng. Mech. Dvn*, ASCE, Vol. 100, No. EM2, April 1974, pp 151-171.

(5) Anderson, JM and Trahair, NS, 'Stability of Monosymmetric Beams and Cantilevers', *Journ. Struct. Dvn*, ASCE, Vol. 98, No. ST1, Jan 1972, pp 269-286.

(6) Kitipornchai, S and Trahair, NS, 'Buckling of Inelastic I-Beams under Moment Gradient', *Journ. Struct. Dvn*, ASCE, Vol. 101, No. ST5, May 1975, pp 991-1004.

(7) Kitipornchai, S and Trahair, NS, 'Elastic Behaviour of Tapered Monosymmetric I-beams', *Journ. Struct. Dvn*, ASCE, Vol. 101, No. ST8, Aug 1975, pp 1661-1678.

(8) Papangelis, JP and Trahair, NS, 'Buckling of Monosymmetric Arches Under Point Loads', *Engineering Structures*, Vol. 10, No. 4, October 1988, pp 257-264.

(9) Cullimore, MSG, 'The Shortening Effect - A Non-Linear Feature Of Pure Torsion', *Engineering Structures*, Butterworths Scientific, London, 1949.

(10) Trahair, NS, 'Non-Linear Elastic Non-Uniform Torsion", *Journal of Structural Engineering*, ASCE, Vol. 131, No. 7, July 2005, pp 1135-42.

(11) Farwell, CR and Galambos, TV, 'Non-Uniform Torsion of Steel Beams in Inelastic Range', *Journ. Struct. Dvn*, ASCE, Vol. 95, No. ST12, December 1969, pp 2813-2829.

(12) Pi, YL, and Trahair, NS, 'Inelastic Torsion of Steel I-Beams', *Journal of Structural Engineering*, ASCE, Vol. 121, No. 4, April 1995, pp 609- 620.

(13) Masur, EF and Milbradt, KP, 'Collapse Strength of Redundant Beams After Lateral Buckling', *Journal of Applied Mechanics*, ASME, Vol. 24, No. 2, June 1957, p 283.

(14) Woolcock, ST and Trahair, NS, 'Post-Buckling of Redundant Rectangular Beams', *Journ. Eng. Mech. Dvn*, ASCE, Vol. 101, No. EM4, Aug 1975, pp 301-316.

(15) Woolcock, ST and Trahair, NS, 'Post-Buckling of Redundant I-Beams', *Journ. Eng Mech. Dvn*, ASCE, Vol. 102, No. EM2, April 1976, pp 293-312.

(16) Trahair, NS, 'Multiple Design Curves for Beam Lateral Buckling', *Proceedings*, 5th International Colloquium on Stability and Ductility of Steel Structures, Nagoya, 1997, pp 33-44.

(17) Trahair, NS, 'Design of Steel Single Angle Lintels', *Journal of Structural Engineering*, ASCE, Vol. 135, No. 5, May 2009, pp 539-45.

APPENDIX II - NOTATION

A	cross-sectional area
a_c, a_t	flange distances from shear centre
a_0	distance to shear centre
b	width
E	Young's modulus of elasticity
e	load eccentricity
F_c, F_t	flange forces
f_y	yield stress
G	shear modulus of elasticity
I_n	Wagner section constant
I_w	warping section constant
I_x, I_y	second moments of area about the x, y axes
J	torsion section constant
L	member length
M	applied moment
M_b	nominal member moment capacity
M_e	elastic buckling moment
M_{max}	maximum moment
M_{px}, M_{py}	full plastic moments about x, y axes
M_{sx},M_{sy}	section moment capacities about x, y axes
M_Y	first yield moment
M_{yz}	elastic buckling moment of a beam in uniform bending
P	axial compression
P_e	elastic buckling load
P_y	minor axis buckling load
Q	concentrated load
r_0	polar radius of gyration
t	thickness
w	axial shortening
x, y	principal axis coordinates
y_0	shear centre coordinate
z	distance along beam
β_x	end moment ratio
ϕ	twist rotation
ϕ_L	end twist rotation
λ_e	modified slenderness
θ	section rotation

FIGURES

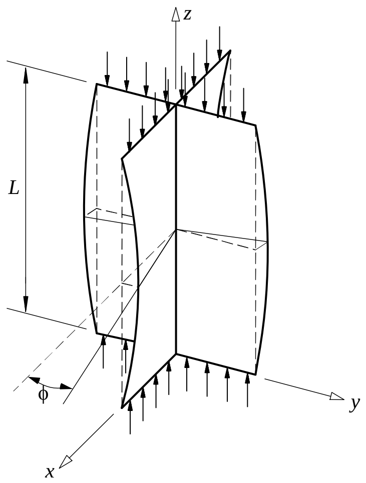


Fig. 1. Torsional Buckling of a Cruciform Section

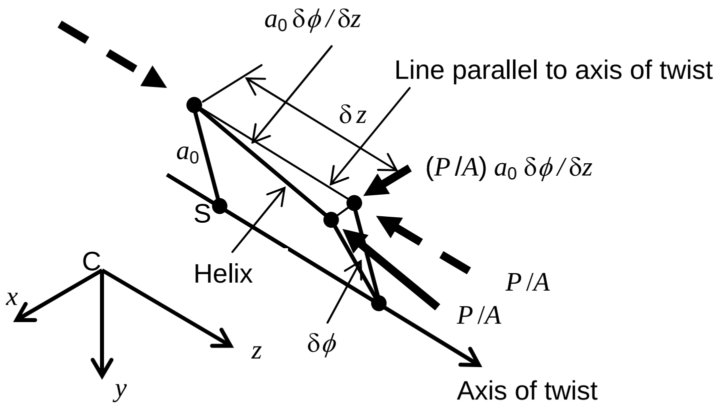
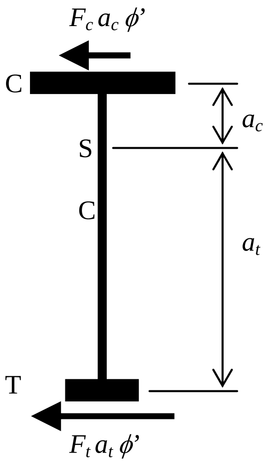


Fig. 2. Torque Exerted by Axial Stresses During Twisting.



$$\boxed{F_c \sim F_t, \quad a_c < a_t}$$

$$\boxed{M\beta_x \phi' = F_t a_t \phi' a_t - F_c a_c \phi' a_c > 0}$$

Fig. 3. Wagner Effect in Monosymmetric Beams.

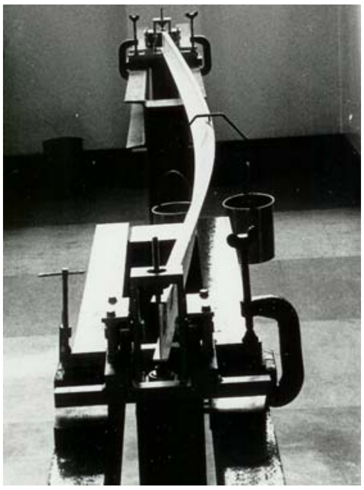


Fig. 4 Buckled Beam

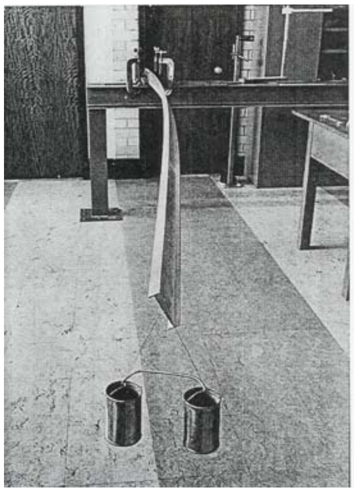


Fig. 5. Buckled Cantilever

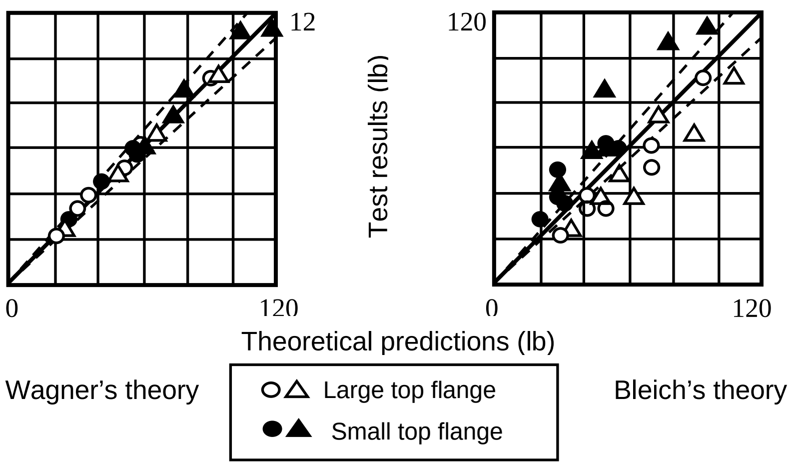


Fig. 6. Analysis and Experiment for Monosymmetric Cantilevers

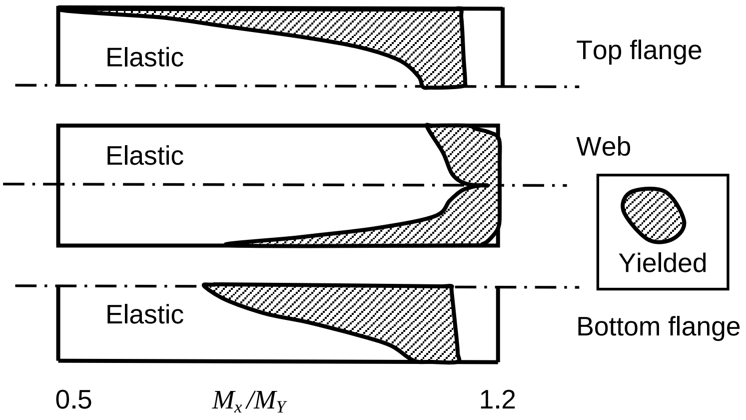


Fig. 7. Yielding of I-Beams Under Moment Gradient

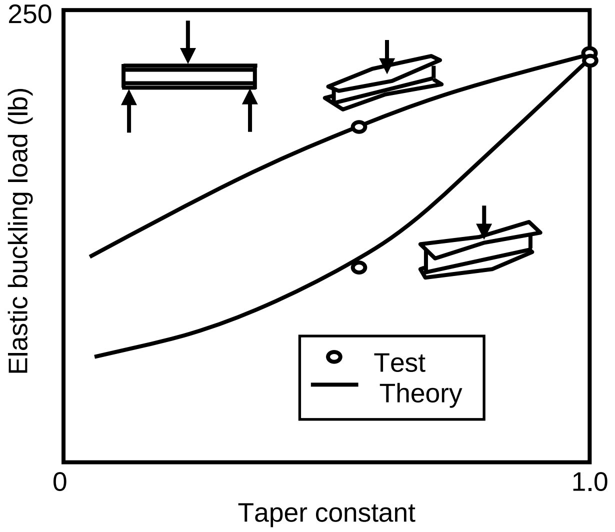


Fig. 8. Buckling of Tapered Monosymmetric Beams.

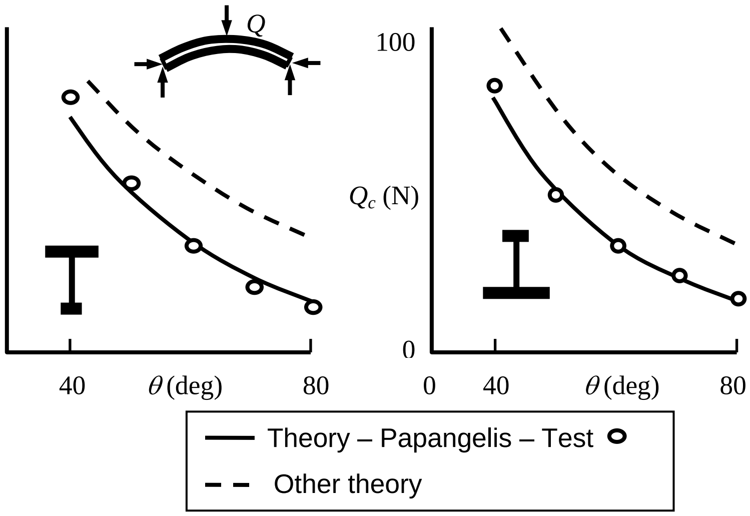


Fig. 9. Buckling of Monosymmetric Arches.

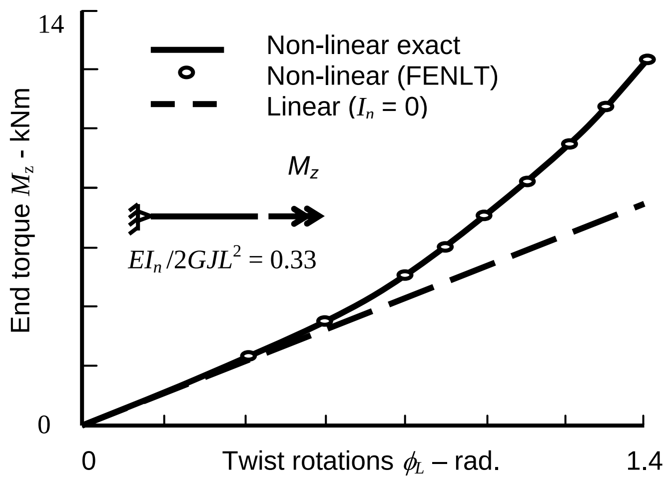


Fig. 10. Large Elastic Twist Rotations of a Rectangular Section

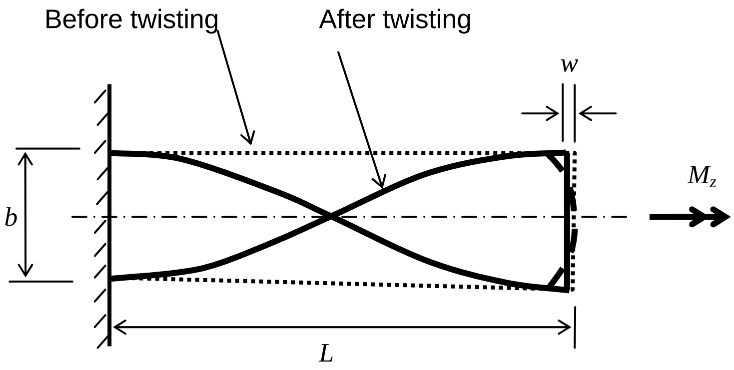


Fig. 11. Axial Shortening of a Rectangular Section Cantilever

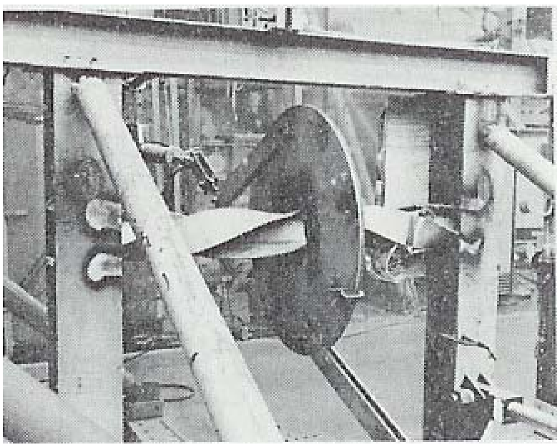


Fig. 12. Inelastic Torsion of an I-Beam.

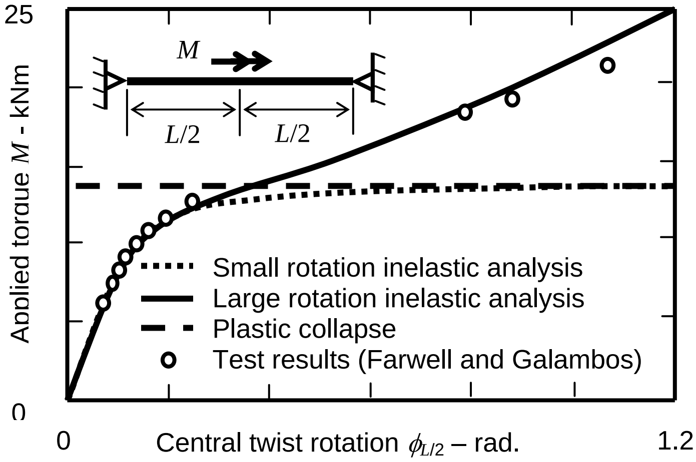


Fig. 13. Inelastic Torsion Test and Theory.

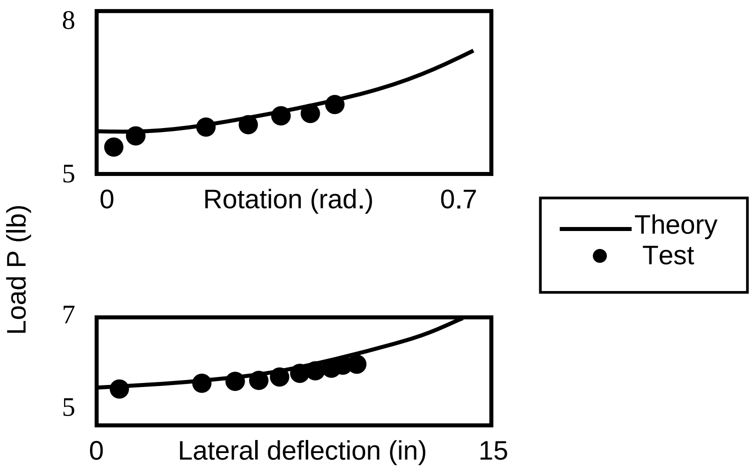


Fig. 14. Post-Buckling of a Cantilever.

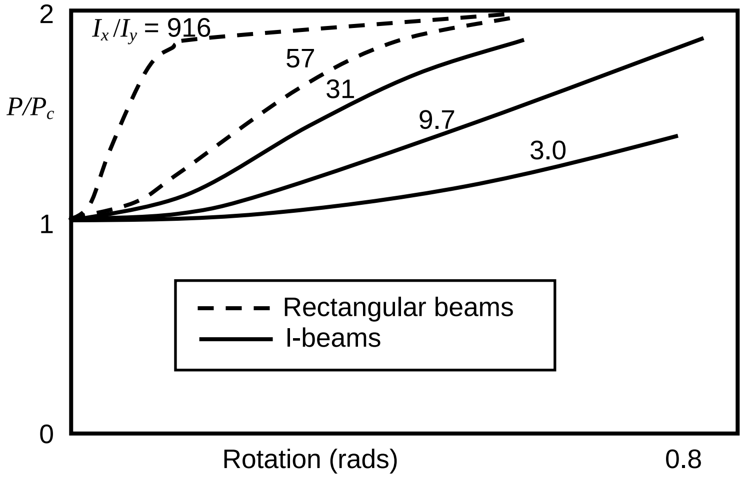


Fig. 15. Post-Buckling of Redundant Beams.

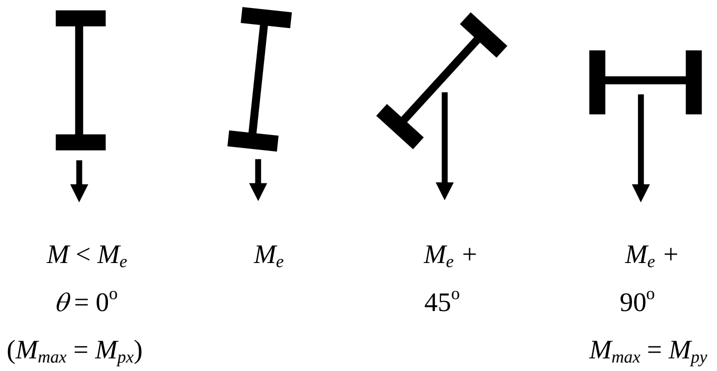


Fig. 16. Large Rotations of an I-Beam.

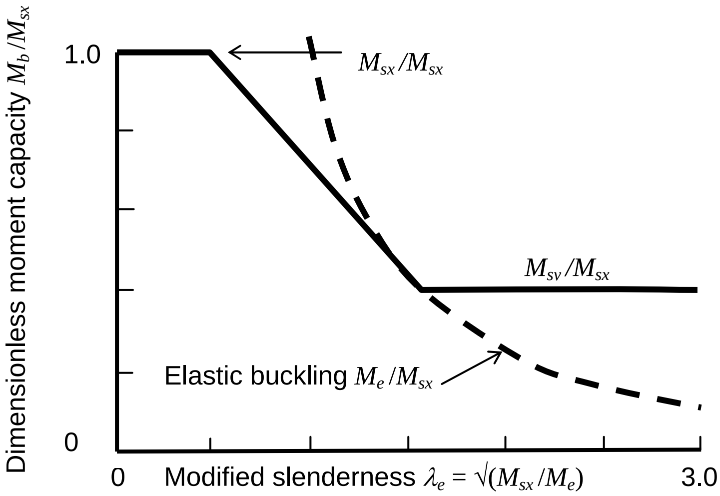
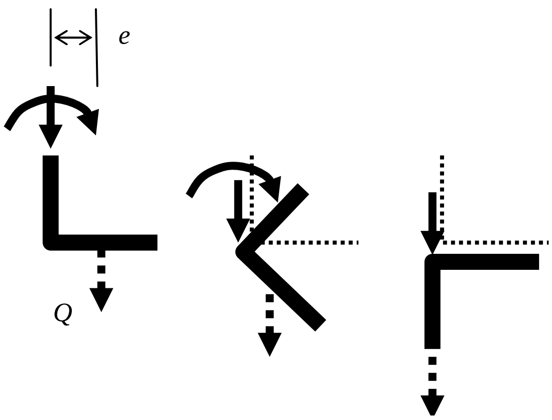
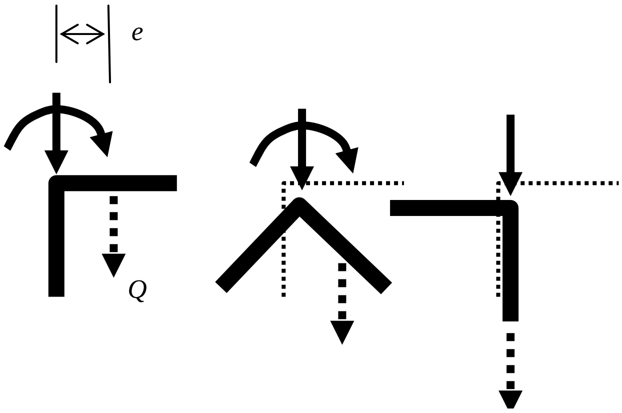


Fig.17. Lateral Buckling Strengths of Steel Beams



	$\phi = 0$	45^o	90^o
Torque	$= Qe$	$Qe/\sqrt{2}$	0
$\dfrac{\text{Moment capacity}}{f_y b^2 t}$	= 0.41	0.71	0.41

Fig. 18. Lintel with Horizontal Leg Down.



	$\phi = 0$	45^o	90^o
Torque	$= Qe$	$Qe / \sqrt{2}$	0
$\dfrac{\text{Moment capacity}}{f_y b^2 t}$	= 0.41	0.35	0.41

Fig. 19. Lintel with Horizontal Leg Up.