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**Shear Buckling of Thin-Walled  
Channel Sections with Intermediate  
Web Stiffener**

**Research Report No R892**

**Cao Hung Pham BE MConstMgt MEngSc  
Gregory J Hancock BSc BE PhD DEng**

**September 2008**

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### **Abstract:**

The elastic buckling stress of the web of a thin-walled section member in shear is generally improved by the presence of flanges and lips. However, for webs with relatively large depth-to-thickness ratios, the local buckling mode in shear occurs mainly in the web. The structural efficiency of such webs can be improved by adding an intermediate stiffener cold-formed longitudinally in the middle of the web.

In this report, the computational modelling of thin-walled steel sections is implemented by means of a spline finite strip analysis to determine the elastic buckling stresses of channel sections subject to pure shear. Lipped channels with an intermediate web stiffener are studied where the main variables are the dimensions of the stiffener in both depth and width directions. Results and comparisons of analyses are included in this report.

### **Keywords:**

Shear buckling; Intermediate Web Stiffener Thin-walled channel sections; Lipped and unlipped channel sections; Spline finite strip method; Shear buckling capacity; Twisting and lateral buckling mode.

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#### Shear Buckling of Thin-Walled Channel Sections with Intermediate Web Stiffener

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# TABLE OF CONTENTS

<b>1</b>	<b>INTRODUCTION .....</b>	<b>4</b>
	<i>1.1 Shear buckling studies of thin-walled sections .....</i>	<i>4</i>
<b>2</b>	<b>MODELLING SECTIONS WITH INTERMEDIATE STIFFENER IN SHEAR...6</b>	<b>6</b>
	<i>2.1 Lipped Channel Geometry with Intermediate Stiffener .....</i>	<i>6</i>
	<i>2.2 Shear Stress Distribution and Boundary Conditions.....</i>	<i>6</i>
<b>3</b>	<b>RESULTS OF BUCKLING ANALYSES .....</b>	<b>8</b>
<b>4</b>	<b>CONCLUSION .....</b>	<b>14</b>
	<b>REFERENCES.....</b>	<b>15</b>

# 1 INTRODUCTION

## 1.1 Shear buckling studies of thin-walled sections

Thin-walled sections can be subjected to axial force, bending and shear. In the cases of axial force and bending, the actions causing buckling, either Euler buckling for compression or flexural-torsional buckling for flexure, are well understood. However, for shear, the traditional approach has been to investigate shear plate buckling in the web alone and to ignore the behaviour of the whole section including the flanges. Until recently, there does not appear to be any consistent investigation of the full section buckling of thin-walled sections under shear.

Recently Pham and Hancock (2007) have provided solutions to the elastic shear buckling of complete channel sections loaded in pure shear parallel with the web by using spline finite strip analysis (Lau and Hancock, 1986) implemented in the program ISFSM Isoparametric Spline Finite Strip Method (Eccher, 2007). Fig 1 from Pham and Hancock (2007) shows the results of the buckling analyses of the lipped channel section of length  $a=1000$  mm and the ratios of flange to web width ( $b_2/b_1$ ) from 0.00005 to 0.8.

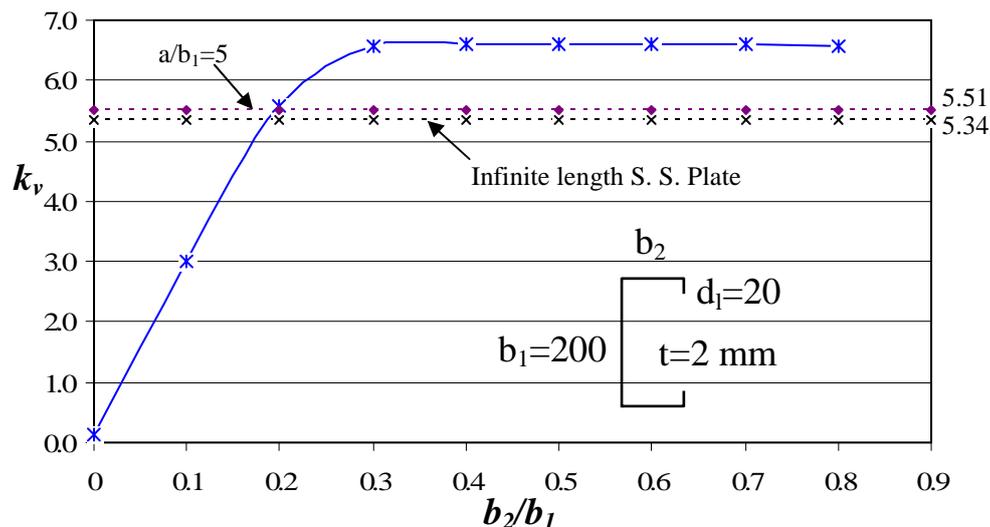


Figure 1. The Ratio of Flange and Web Widths ( $b_2/b_1$ ) and The Shear Buckling Coefficients ( $k_v$ )

The coefficient  $k_v$  in Fig 1 is the shear buckling coefficient in the expression for the elastic shear buckling stress generalized by Timoshenko and Gere (1961) in the following equation:

$$\tau_{cr} = k_v \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t}{b_1} \right)^2$$

where  $E$  = modulus of elasticity;  $\nu$  = Poisson's ratio;  $b_1$  = width of the plate;  $t$  = thickness of the plate.  $k_v$  is the shear buckling coefficient, which depends on the boundary conditions and the aspect ratio of the rectangular plate  $a/b_1$ . In the development of Fig 1, the shear flow is that for a channel with a force parallel with the web through the shear centre. All edges of the end cross-section are simply supported and there are no lateral restraints along two longitudinal edges of the web panel. The corresponding buckling mode shapes are shown in Fig 2.

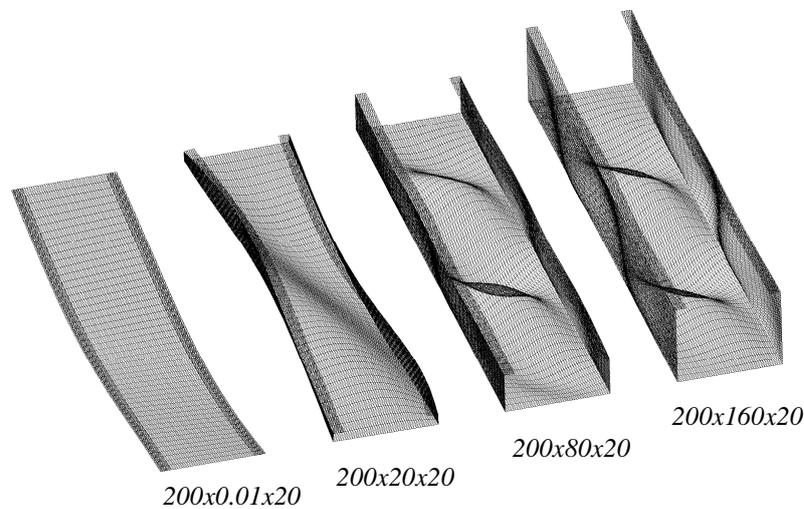


Figure 2. Buckling Mode Shape of Lipped Channel Section – Length = 1000 mm,  $a/b_1=5$

The analysis results show that the lack of lateral restraint for sections with narrow flanges can lead to premature buckling of the section in a twisting and lateral buckling mode. As the ratio of  $b_2/b_1$  increases to 0.4, the value of  $k_v$  increases rapidly to 6.61 which is greater than the theoretical result of 5.34 for a simply supported rectangular plate in shear of infinite length (Timoshenko and Gere, 1961; Bulson, 1970; Bleich, 1952; Allen and Bulson, 1980) and of 5.51 for a simply supported rectangular plate of length to width ratio of 5 (Pham and Hancock, 2007). This is apparently due to the fact that the flanges can have a significant influence on improving the shear buckling capacity of thin-walled channel sections. The flanges with lips are long enough to give full lateral restraints to the lipped channel section members. For the ratio of  $b_2/b_1$  ranging from 0.4 to 0.8, the value of  $k_v$  reduces slightly to 6.57. The buckling modes shown in Fig 2 are mainly local buckling modes in the web and the flanges. The explanation for the slight reduction of  $k_v$  is due to the effect of the slenderness of the wider flange and buckling in the flange as shown in Fig 2 for the 200x160x20 section.

## 2 MODELLING SECTIONS WITH INTERMEDIATE STIFFENER IN SHEAR

### 2.1 Lipped Channel Geometry with Intermediate Stiffener

The geometry of the lipped channel with an intermediate stiffener studied in this report is shown in Fig 3. The channel section consists of a web of width 200 mm ( $b_1$ ), a flange of width 80 mm ( $b_2$ ), a lip size of 20 mm, all with thickness of 2 mm. The stiffener is positioned at the longitudinal center line of the web and the main variables are the dimensions of the stiffener. The depth of stiffener ( $b_{1s}$ ) increases from 0.01 mm to 160 mm whereas the width of the stiffener ( $b_{2s}$ ) varies from 0.05 mm to 50 mm. The member is subdivided into 40 longitudinal strips which include 12 strips in the web, 6 strips in each flange and 2 strips in each lip. The strip subdivision of the stiffener is 6 strips and 2 strips in the depth and width respectively. The length of the member studied is 1000 mm. The aspect ratio of the web rectangular plate is therefore  $a/b_1 = 5$ .

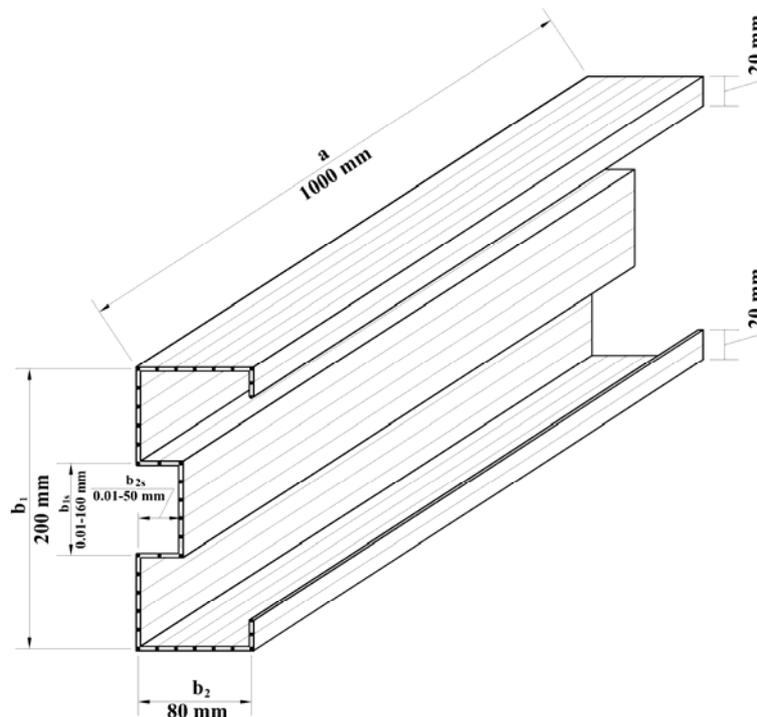


Figure 3. Lipped Channel Geometry with an Intermediate Stiffener

### 2.2 Shear Stress Distribution and Boundary Conditions

In order to demonstrate the different ways in which a channel member with a variable size intermediate web stiffener may buckle under shear stress, the shear stress distribution in the complete channel section is firstly modelled. The shear flow distribution resulting from a shear force parallel with the web is shown in

Fig 4. To simulate the variation in shear stress with the spline finite strip analysis, each strip in the cross-section is assumed to be subjected to a pure shear stress which varies from one strip to the other. The more the cross-section is subdivided into strips, the more accurately the shear stress is represented in order to match the practical shear flow distribution. The spline finite strip analysis as implemented in the Lau and Hancock theory does not allow variation in shear flow across the width of a strip.

In this report, the boundary conditions of the end cross-section assume all edges are simply supported. There are no lateral restraints along the two longitudinal edges of web panels and stiffener. Fig 5 shows the boundary conditions of the lipped channel with an intermediate web stiffener.

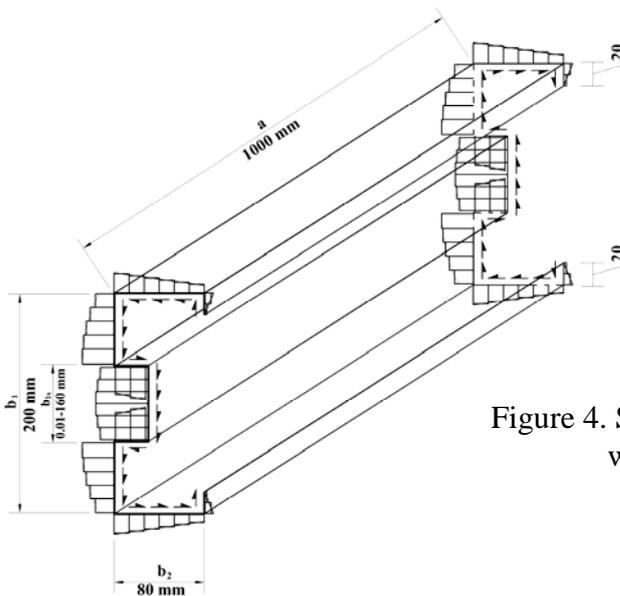


Figure 4. Shear Flow Distribution in Lipped Channel with an Intermediate Web stiffener

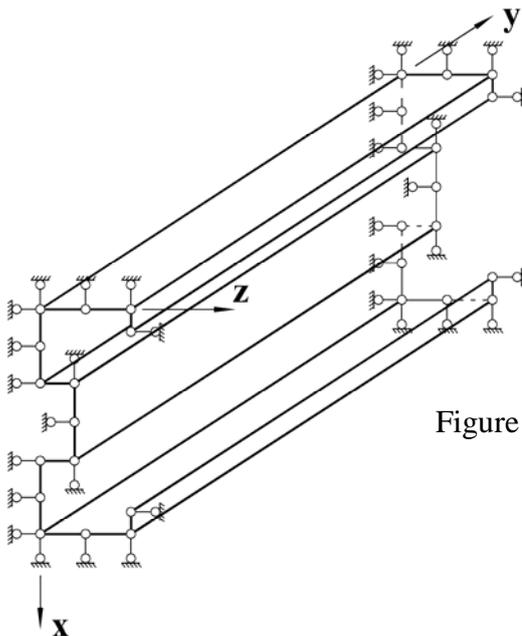


Figure 5. Boundary Conditions of Lipped Channel with an Intermediate Web Stiffener

### 3 RESULTS OF BUCKLING ANALYSES

$b_{1s}/b_1$	$b_{2s}/b_1$										
	0.00	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
<b>0.0</b>	6.59	7.40	8.78	10.46	12.57	14.04	15.50	16.84	17.83	18.37	18.40
<b>0.1</b>	6.59	9.22	12.05	15.25	18.06	21.05	23.46	25.44	27.07	27.87	28.14
<b>0.2</b>	6.59	10.23	14.71	18.62	23.02	26.12	28.68	30.75	32.36	33.53	33.30
<b>0.3</b>	6.59	10.72	15.69	19.94	25.16	30.34	32.92	34.74	35.96	36.73	36.69
<b>0.4</b>	6.59	10.59	15.16	18.54	22.38	26.20	29.63	32.47	34.60	35.47	34.92
<b>0.5</b>	6.59	10.02	13.81	16.15	18.73	21.33	23.69	24.30	23.93	23.58	23.25
<b>0.6</b>	6.59	9.30	12.28	13.95	15.79	17.59	17.74	17.47	17.22	16.98	16.76
<b>0.7</b>	6.59	8.56	10.65	11.96	13.19	13.65	13.49	13.31	13.14	12.98	12.82
<b>0.8</b>	6.59	7.72	8.93	10.08	10.72	10.77	10.68	10.57	10.46	10.35	10.25

Table 1. Shear Buckling Coefficient ( $k_v$ ) of Lipped Channel Sections with Intermediate Web Stiffener

The results of the buckling analyses of the lipped channel section with an intermediate web stiffener are shown in Table 1. Fig 6 shows the relationship between the ratio of stiffener depth and web width ( $b_{1s}/b_1$ ) from 0.00005 to 0.8 ( $b_{1s} = 0.01-160$  mm) and the shear buckling coefficients ( $k_v$ ). Each relationship curve represents a different stiffener width ( $b_{2s}$ ) which is in the range from 0.01 mm to 50 mm.

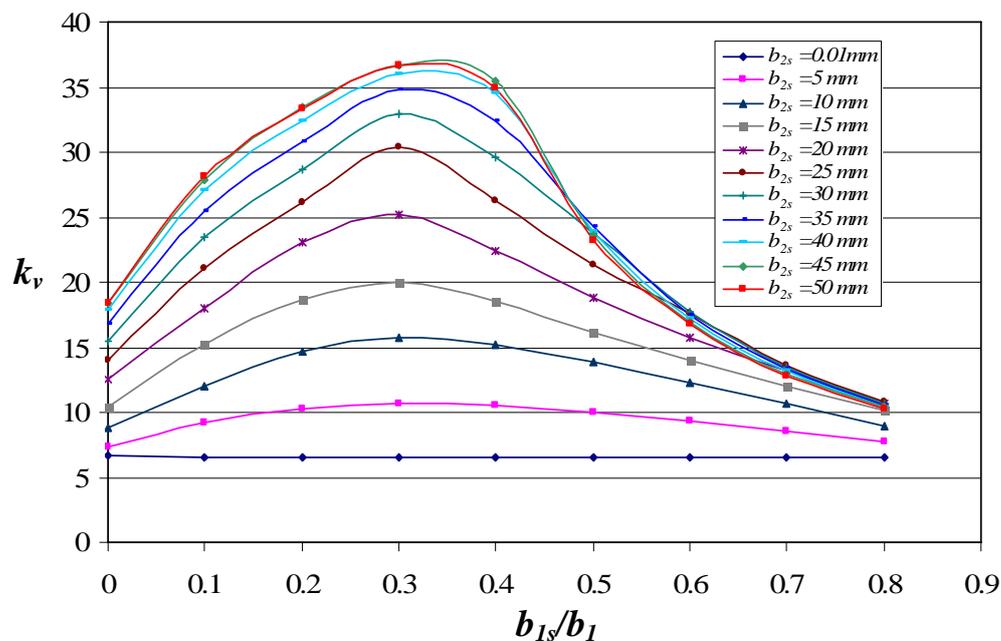


Figure 6. The Ratio of Stiffener Depths and Web Widths ( $b_{1s}/b_1$ ) and The Buckling Coefficients ( $k_v$ ) Of Lipped Channel Section with an Intermediate Web Stiffener

As can be seen in Table 1, when the stiffener width ( $b_{2s}$ ) is very small ( $b_{2s} = 0.01$  mm), the value of  $k_v$  stays unchanged (6.59) while the stiffener depth varies from 0.01 mm to 160 mm. This case is almost exactly the same as that of the plain channel section 200x80x20 shown in Fig 2. Therefore, the buckling coefficient curve ( $k_v$ ) with  $b_{2s} = 0.01$  mm is a horizontal line in Fig 6. It can be noted that the buckling mode shown in Fig 2 for this case is a local buckling mode which mainly occurs in the web. The flanges are long enough to provide elastic torsional restraint on the web. There is no distortional buckling mode in the flanges.

For a stiffener width ( $b_{2s}$ ) of 5 mm, when the ratio of ( $b_{1s}/b_1$ ) is 0.00005, the value of  $k_v$  is 7.40 which is slightly greater than that of a plain channel section (6.59). As the ratio of ( $b_{1s}/b_1$ ) increases to 0.3, the value of  $k_v$  improves to 10.72. This can be explained by the fact that the deeper stiffener can contribute to the shear buckling capacity of the channel section member. However, it is interesting to note that when the ratio of ( $b_{1s}/b_1$ ) increases further from 0.3 to 0.8, the value of  $k_v$  reduces from 10.72 to 7.74. This is due to the slenderness of longer stiffener depth which allows shear buckling in its own length. As the stiffener depth ( $b_{1s}$ ) gets longer (60 mm to 160 mm), the slenderness is more critical.

Fig 7 shows the corresponding buckling mode shapes for different stiffener depths ( $b_{1s}$ ) from 0.01 mm to 160 mm for lipped channel section with an intermediate web stiffener for the case of  $b_{2s}$  equal to 5 mm. When the stiffener depth ( $b_{1s}$ ) is 0.01 mm, the local buckling mode, which occurs mainly in the web, is almost similar to that of the plain channel section shown in Fig 2. There is little distortional buckling mode in the lipped flanges. As the stiffener depth ( $b_{1s}$ ) increases to 60 mm, local buckling still occurs in the web but is not as clear as that for the stiffener depth of  $b_{1s}$  equal to 0.01 mm. The flanges with lips start buckling in the distortional buckling mode. This is mainly due to the fact that the presence of a longer stiffener depth ( $b_{1s}$ ) improves significantly the shear buckling capacity which causes not only local buckling in the web but also distortional buckling of the flanges and lips at higher shear load. When the stiffener depth ( $b_{1s}$ ) is in the range from 60 mm to 160 mm, the buckling mode is mainly in the stiffener depth ( $b_{1s}$ ). The distortional buckling in the flanges and lips become less severe which explains the reduction of the value of  $k_v$  when the ratio of ( $b_{1s}/b_1$ ) is in the range from 0.3 to 0.8.

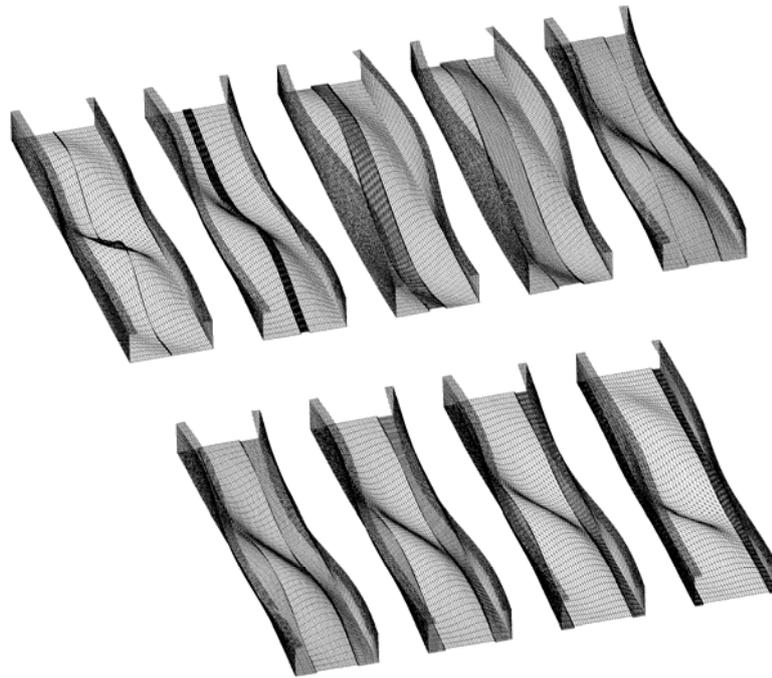


Figure 7. Buckling Mode Shape of Lipped Channel Section  
With Intermediate Web Stiffener –  $b_{2s} = 5 \text{ mm}$

As can also be seen Fig 6, the relationship curves between the ratio of stiffener depths and web width ( $b_{1s}/b_1$ ) and the shear buckling coefficients ( $k_v$ ) behave similarly to that for the ratio of  $b_{1s}/b_1$  equal to 0.00005 when the stiffener width ( $b_{2s}$ ) increases from 0.01 mm to 50 mm. The value of  $k_v$  also increases until the ratio of ( $b_{1s}/b_1$ ) reaches 0.3. The value of  $k_v$  then reduces as the ratio of ( $b_{1s}/b_1$ ) increases further from 0.3 to 0.8. It can be seen in Fig 6 that when the stiffener width ( $b_{2s}$ ) increases from 0.01 mm to 25 mm, the relationship curves between the ratio of  $b_{1s}/b_1$  and the shear buckling coefficients ( $k_v$ ) increase with the stiffener width ( $b_{2s}$ ). The increments of the shear buckling coefficients ( $k_v$ ) are significantly greater at smaller values of the stiffener width ( $b_{2s}$ ) and decrease in the range of the stiffener width ( $b_{2s}$ ) from 25 mm to 50 mm. Further, when the ratio of  $b_{1s}/b_1$  increases from 0.5 to 0.8, the value of  $k_v$  drops more significantly with the ratio of  $b_{1s}/b_1$  for larger values of the stiffener width ( $b_{2s}$ ). The shear buckling coefficients ( $k_v$ ) lie slightly below those of the stiffener width ( $b_{2s}$ ) of 35 mm as the stiffener width ( $b_{2s}$ ) is in the range from 35 mm to 50 mm. The explanation is mainly a result of the effect of slenderness in longer stiffener width ( $b_{2s}$ ). The longer the stiffener is, the more significantly the shear buckling capacity is affected so that for longer stiffener widths ( $b_{2s}$ ), the slenderness is critical. Local buckling also occurs in the stiffener width ( $b_{2s}$ ) which causes a significant reduction of the shear buckling capacity of the full channel section.

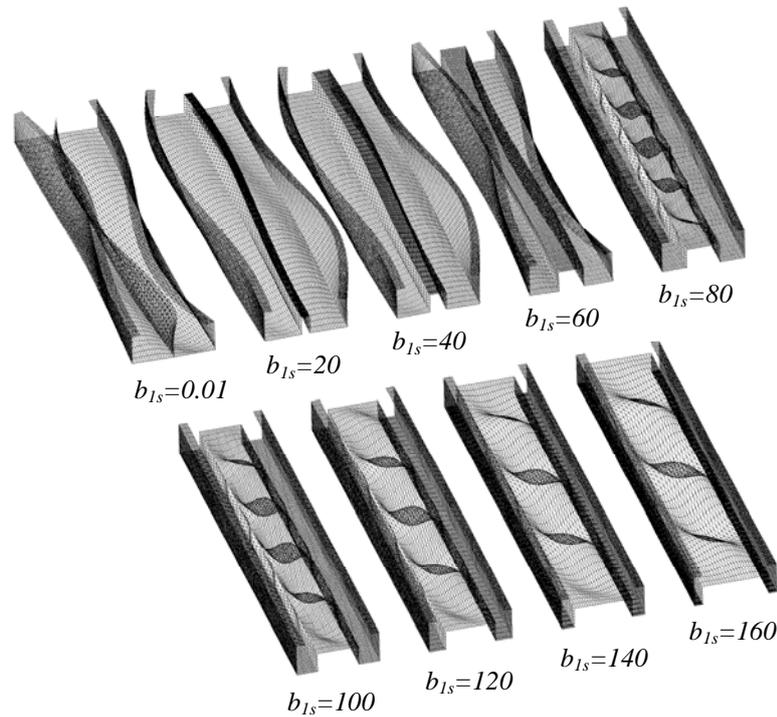


Figure 8. Buckling Mode Shape of Lipped Channel Section with Intermediate Web Stiffener –  $b_{2s} = 50 \text{ mm}$

Fig 8 shows the corresponding buckling mode shapes of different stiffener depths ( $b_{1s}$ ) from 0.01 mm to 160 mm of the lipped channel section with an intermediate web stiffener for the case ( $b_{2s}$ ) of 50 mm. As the stiffener depths ( $b_{1s}$ ) increases to 60 mm, local buckling in the web becomes less critical and the buckling mode is mainly a distortional buckling mode in the flanges. For the stiffener depths ( $b_{1s}$ ) from 80 mm to 160 mm, the buckling mode is mainly local buckling in the stiffener depth and width. The line junctions of the stiffener depth and width remain almost straight. As shown in Fig 8, there is little or no local buckling in the original flat part of the webs and little or no distortional buckling of the flanges of the full channel section

To further understand the behaviour of the shear buckling coefficients ( $k_v$ ) of the channel section with an intermediate web stiffener where the main variables are the stiffener depth ( $b_{1s}$ ) and width ( $b_{2s}$ ), Fig 9 shows the relationship between the ratio of stiffener width and web width ( $b_{2s}/b_1$ ) from 0.00005 to 0.25 ( $b_{2s} = 0.01\text{-}50 \text{ mm}$ ) and the shear buckling coefficients ( $k_v$ ). Each relationship curve represents a different stiffener depth ( $b_{1s}$ ) which is in the range from 0.01 mm to 160 mm.

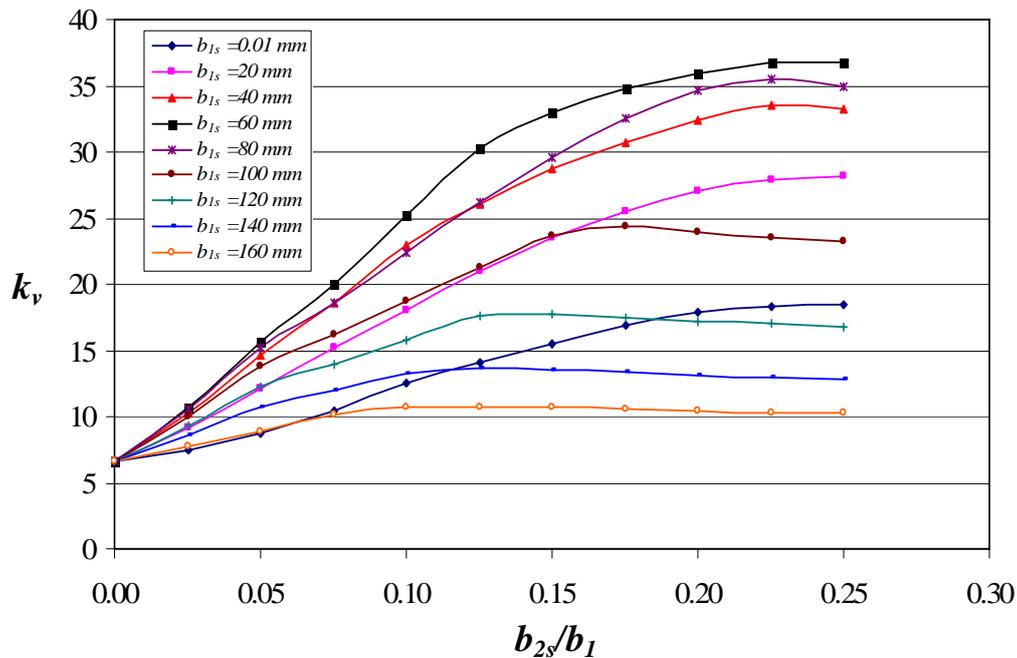


Figure 9. The Ratio of Stiffener Widths and Web Widths ( $b_{2s}/b_1$ ) and The Buckling Coefficients ( $k_v$ ) Of Lipped Channel Section with Intermediate Web Stiffener

It can be seen in Fig 9 that when the stiffener width ( $b_{2s}$ ) is 0.01 mm, the value of  $k_v$  is 6.59 irrespective of the stiffener depth. For a stiffener depth ( $b_{1s}$ ) of 20 mm, the value of  $k_v$  increases dramatically from 6.59 to 28.14. As can be seen in Fig 10 which shows the corresponding buckling mode for the case of stiffener depth ( $b_{1s}$ ) of 20 mm with increasing stiffener width ( $b_{2s}$ ) from 0.01mm to 50 mm, when the stiffener width ( $b_{2s}$ ) is 0.01 mm, the buckling mode is mainly local buckling in the web. There is little distortional buckling in the lipped flanges when the stiffener width ( $b_{2s}$ ) is 0.01 mm. As the stiffener width ( $b_{2s}$ ) increases to 50 mm, the local buckling in the web is less critical while there is more distortional buckling in the lipped flanges. The explanation is mainly due to fact that the presence of the longer stiffener width ( $b_{2s}$ ) improves significantly the shear buckling capacity of the web.

As can also be seen in Fig 9, when the stiffener depth ( $b_{1s}$ ) increases to 60 mm, the relationship curves between the ratio of stiffener widths and web width ( $b_{2s}/b_1$ ) and the shear buckling coefficients ( $k_v$ ) behave similarly to those for the stiffener depth ( $b_{1s}$ ) of 20 mm although the shear buckling coefficients ( $k_v$ ) increases more rapidly with  $b_{2s}/b_1$ . However, it is interesting to note that the relationship curves are less straight and the increments are less when the stiffener depth ( $b_{1s}$ ) approaches 60 mm. This can be explained mainly by the effect of the slenderness of the stiffener width ( $b_{2s}$ ) which allows more distortional buckling in the wider stiffener. It is also interesting to note in Fig 9 that when the stiffener depth ( $b_{1s}$ ) increases further from 60 mm to 160 mm, the latter relationship curves lie below the former ones. As discussed earlier, this is mainly due to distortional buckling in the the wider stiffener.

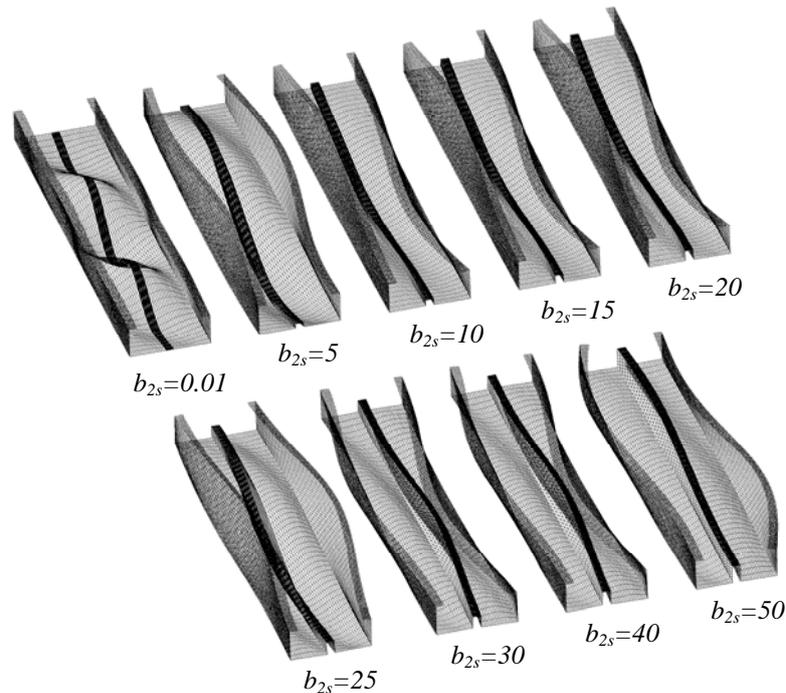


Figure 10. Buckling Mode Shape of Lipped Channel Section  
With Intermediate Web Stiffener –  $b_{1s} = 20 \text{ mm}$

Fig 11 shows the corresponding buckling mode shapes for different stiffener widths ( $b_{2s}$ ) from 0.01 mm to 50 mm of the lipped channel section with an intermediate web stiffener for the case ( $b_{1s}$ ) of 160 mm. As the stiffener width ( $b_{2s}$ ) is 0.01 mm, the member in this case is almost exactly the same as that of plain channel section 200x80x20 shown in Fig 2. The buckling mode is mainly local buckling in the web. There is no or little distortional buckling in the lipped flanges. It is interesting to note that when the stiffener width ( $b_{2s}$ ) increases from 0.01 mm to 15 mm, the local buckling in the web is less critical and there is more distortional buckling in the flanges.

As the stiffener width ( $b_{2s}$ ) increases further from 20 mm to 50 mm, the member buckles mainly in the stiffener depth ( $b_{1s}$ ) in the local buckling mode. There is little or no distortional buckling in the lipped flanges and original flat part of the web. This is obviously due to the fact that when stiffener depth is longer, the greater shear stress is distributed in the stiffener depth due to the shear flow. This fact causes the reduction of the shear buckling stress of the channel section member and the buckling mode is mainly local buckling in the longer stiffener depth.

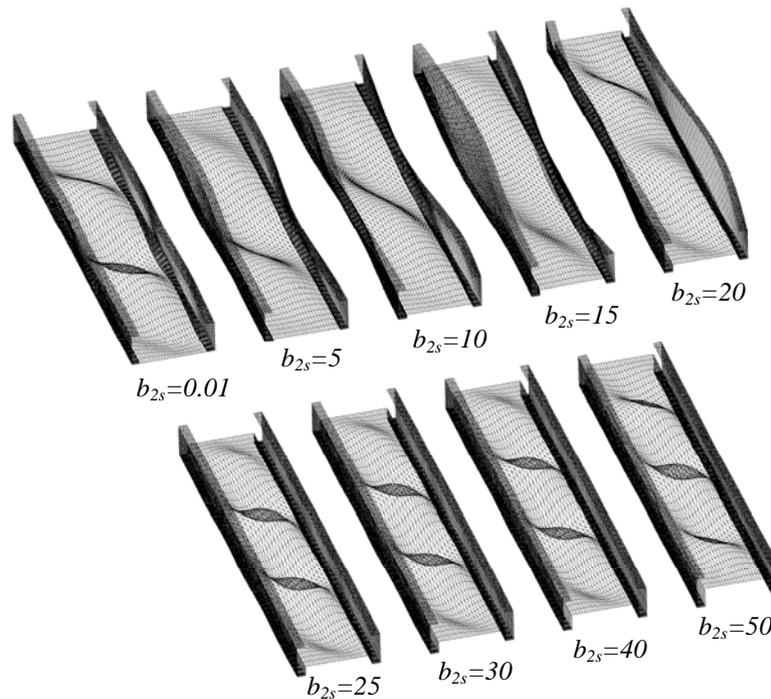


Figure 11. Buckling Mode Shape of Lipped ChanWel Section with Intermediate Web Stiffener –  $b_{1s} = 160 \text{ mm}$

## 4 CONCLUSION

This report has outlined buckling analyses of channel section members subject to shear stresses. Lipped channels with a variable size intermediate web stiffener was analyzed by the Isoparametric Spline Finite Strip Method program. The main variables are the dimensions of the stiffener in both depth and width directions. The boundary conditions are simply supported without lateral restraints along two longitudinal edges of web panel and line junctions of the stiffener. The assumed shear flow distribution in the whole channel section member subject to pure shear parallel with the web is used to investigate the effect of stiffener size on the shear buckling stresses.

By varying the stiffener sizes in both width and depth directions, the analysis results show that the stiffener can have a significant influence on improving the shear buckling stress of thin-walled channel sections up to a certain ratio of stiffener depth to web. The stiffener also reduces local buckling which occur mainly in the web width. Moreover, it is also demonstrated that with longer stiffener depth and wider stiffener width the shear buckling capacity of the lipped channel section with an intermediate web stiffener reduces and the buckling mode is then mainly local buckling mode in longer stiffener depth with increasing stiffener width.

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