AS/NZS 4600. This mode is sometimes called 'Flange Distortional' and is described in Section 5.3.1 of this book. In some cases, such as for the Hollow Flange Beam and the LiteSteel beam subject to bending, distortional buckling may involve transverse flexure of the web as shown in Fig. 3.15 and 3.16 and this is specified in Clause 3.3.3.3(b) of AS/NZS 4600. This mode is sometimes called 'Lateral Distortional' and is described in Section 5.3.2 of this book.

The basic behaviour of purlins is described in Section 5.4, and design methods for purlins are described in Section 5.5. These include the R-factor design approach in Clause 3.3.3.4 of AS/NZS 4600 which allows for the restraint from sheeting attached by screw-fastening to one flange. Methods for bracing beams against lateral and torsional deformation are described in Clause 4.3 of AS/NZS 4600 and are described in Section 5.6 of this book. Allowance for inelastic reserve capacity of flexural members is included as Clause 3.3.2.3 of AS/NZS 4600 as an alternative to initial yielding described by Clause 3.3.2.2 and specified by Eq 5.1. Inelastic reserve capacity is described in Section 5.7 of this book.

The design for (d) requires computation of the nominal shear capacity (V_V) of the beam and is fully described in Chapter 6 (Webs) of this book. The design for (e) requires computation of the nominal capacity for concentrated load (Rb) (bearing) and is also described in Chapter 6 (Webs) of this book. The interaction of both shear and bearing with section moment is also described in Chapter 6.

5.2 Flexural-Torsional (Lateral) Buckling

5.2.1 Elastic Buckling of Unbraced Simply Supported Beams

The elastic buckling moment (M_o) of a simply supported and I-beam, monosymmetric I-beam or T-beam bent about the *x*-axis perpendicular to the web as shown in Fig. 5.1(a) with equal and opposite end moments and of unbraced length (*l*) is given in Refs 5.1 and 5.2 and is equal to:

$$M_o = \frac{\pi\sqrt{EI_y GJ}}{l}\left[\frac{\pi\delta}{2}+\sqrt{\left(\frac{\pi\delta}{2}\right)^2+\left(1+\frac{\pi^2 EI_w}{GJl^2}\right)}\right] \qquad (5.4)$$

where

$$\delta = \frac{\pm\beta_x}{l}\sqrt{\frac{EI_y}{GJ}} \qquad (5.5)$$

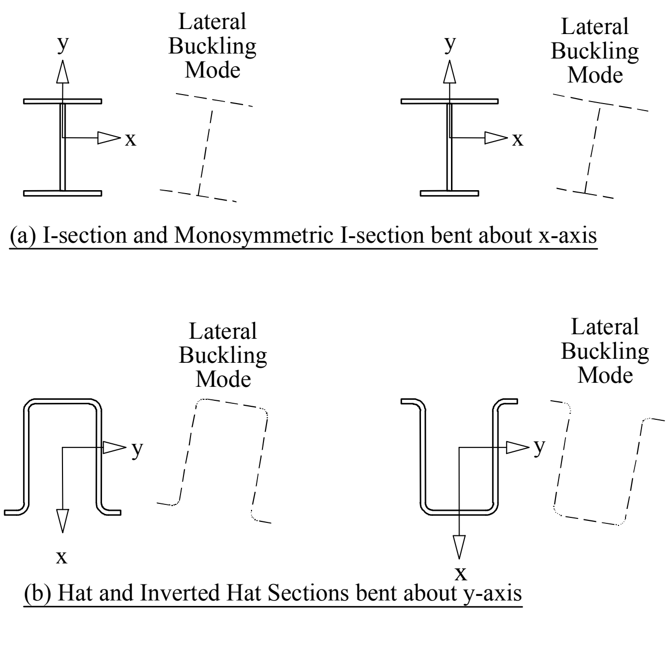


Fig. 5.1 Lateral buckling modes and axes

The value of δ is positive when the larger flange is in compression, is zero for doubly symmetric beams, and is negative when the larger flange is in tension.

The monosymmetry parameter (β_x) is a cross-sectional parameter defined by

$$\beta_x = \frac{\left|\int_A \left(x^2 y + y^3\right) dA\right|}{I_x} - 2y_o \tag{5.6}$$

Formulae to evaluate β_x for a range of thin-walled cross-sections are given in Appendix E2 of AS/NZS 4600.

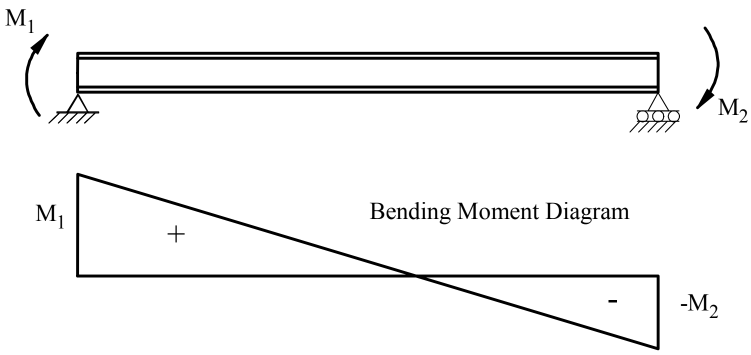
In the case of doubly-symmetric beams, β_x is zero and Eq. (5.4) simplifies to

$$M_o = \frac{\pi\sqrt{EI_y GJ}}{l}\sqrt{1+\frac{\pi^2 EI_w}{GJl^2}} \tag{5.7}$$

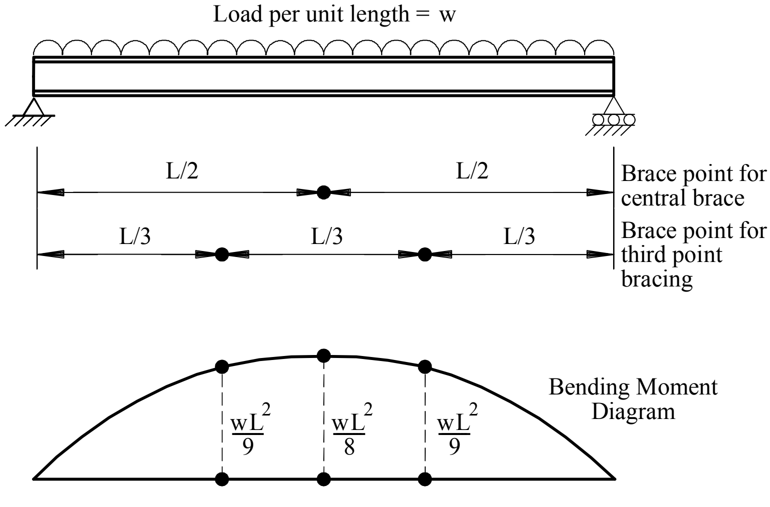
In the case of simply supported beams subjected to non-uniform moment, Eq. (5.7) can be modified by dividing by the factor C_{TF} which allows for the nonuniform distribution of bending moment in the beam.

$$M_o = \frac{\pi\sqrt{EI_y GJ}}{C_{TF}\, l}\sqrt{1+\frac{\pi^2 EI_w}{GJl^2}} \tag{5.8}$$

For a beam subjected to a clockwise moment (M_1) at the left hand end and a clockwise moment (M_2) at the right hand end where M_1 is less than or equal to M_2, as shown in Fig. 5.2(a), then a simple approximation for C_{TF}, as given in Refs 5.1 and 8.1 is



(a) Beam under moment gradient



(b) Beam under uniformly distributed load

Fig. 5.2 Simply supported beams

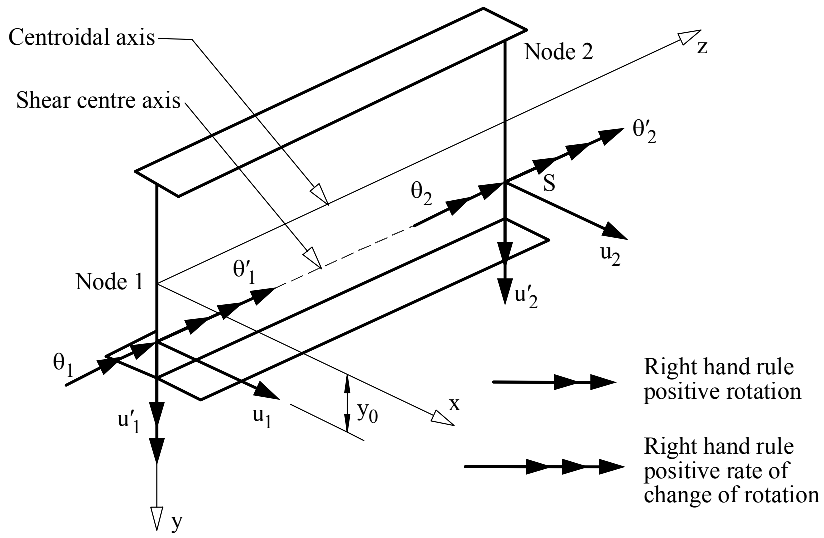
$$C_{TF} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \quad (5.9)$$

The torsion constant for a thin-walled open section is given by

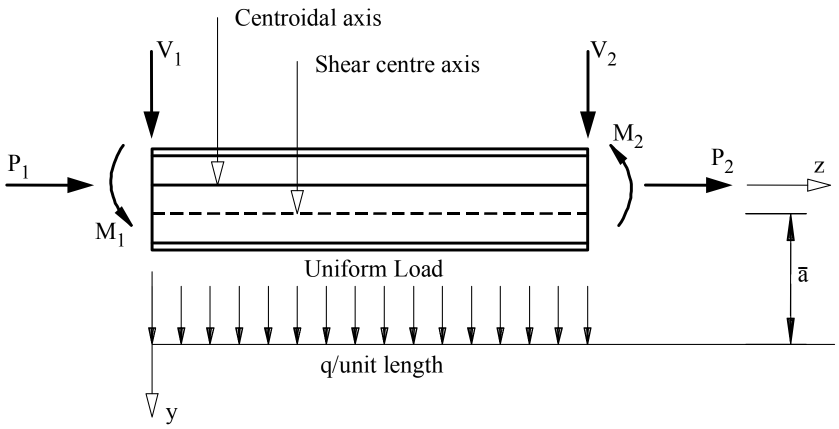
$$J = \frac{\sum bt^3}{3} = \frac{w_f t^3}{3} \quad (5.10)$$

where the summation is taken over all of the elements forming the cross-section or alternatively the feed width of the flat sheet (w_f). For cold-formed sections roll-formed from thin strip, t is very small and hence t^3 is small. The torsional rigidity (GJ) is therefore small compared with the warping rigidity (EI_w / l^2) for practical lengths (l) and hence Eq. (5.8) can be simplified to

$$M_o = \left(\frac{1}{C_{TF}}\right)\left(\frac{\pi^2 E\sqrt{I_y I_w}}{l^2}\right) \quad (5.11)$$



(a) Nodal displacements producing out-of-plane deformation



(b) Element actions in the plane of the structure

Fig. 5.3 Element displacements and actions

The value of Iy to be used for Z sections in Clause 3.3.3.2.1(b) is taken as the value computed about the inclined minor principal axis. Alternatively, for Z sections restrained against lateral movement by sheeting effectively attached to the tension flange, the values of Iy and Iw may be taken as those for an equivalent channel where the direction of the flange of the Z-beam attached to the sheeting is reversed. The justification for this latter approach was given in Ref. 5.3.

The elastic buckling moments (Mo) in Clause 3.3.3.2.1(a) of AS/NZS 4600 are expressed in terms of the elastic buckling stresses (fox, foy and foz). The elastic buckling stresses (fox, foy and foz) for an axially loaded compression member are fully described in Chapter 7 of this book.

For example, Eq. 3.3.3.2(13) of AS/NZS 4600 gives the elastic buckling moment (Mo) for a singly symmetric section bent about the centroidal y-axis perpendicular to the symmetry x-axis such as the hat section in Fig. 5.1(b), as follows:

$$M_o = C_s A f_{ox} \frac{\left[\left(\frac{\beta_y}{2}\right) + C_s\sqrt{\left(\frac{\beta_y}{2}\right)^2 + r_{o1}^2\left(\frac{f_{oz}}{f_{ox}}\right)}\right]}{C_{TF}} \quad (5.12)$$

By substitution of f_{ox} and f_{oz} from Eqs 3.3.3.2(14) and (12) of AS/NZS 4600, Eq. (5.12) becomes

$$M_o = \pi \frac{\sqrt{EI_x GJ}}{l_{ex}} \frac{\left[\frac{\pi\delta}{2} + \sqrt{\left(\frac{\pi\delta}{2}\right)^2 + \left(1 + \frac{\pi^2 EI_w}{GJl_{ez}^2}\right)}\right]}{C_{TF}} \quad (5.13)$$

where $$\delta = \frac{\pm\beta_y}{l_{ex}}\sqrt{\frac{EI_x}{GJ}} \quad (5.14)$$

This is the same as Eqs (5.4) and (5.5) except that:

(i) the *x*-axis is the axis of symmetry and the beam is bent about the *y*-axis, and

(ii) the C_{TF} factor to allow for non-uniform moment is included as in Eq. (5.8), and

(iii) the unbraced length (l) is replaced by l_{ex} for the unbraced flexural length about the *x*-axis, and l_{ez} for the unbraced torsional length, as appropriate.

Hence Eq. 3.3.3.2(13) in Clause 3.3.3.2.1(a)(ii) of AS/NZS 4600 is a more general version of Eqs (5.4) and (5.5) with the *x*- and *y*-axes interchanged.

As a second example, Eq. 3.3.3.2(8) of AS/NZS 4600 gives the elastic buckling moment (Mo) for a singly-symmetric section bent about the symmetry axis, doubly-symmetric sections bent about the x-axis and for Z-sections bent about an axis perpendicular to the web, as follows:

$$M_o = C_b A r_{o1}\sqrt{f_{oy} f_{oz}} \quad (5.15)$$

By substitution of f_{oy} and f_{oz} from Eqs 3.3.3.2(11) and 3.3.3.2(12) of AS/NZS 4600, Eq. (5.15) becomes:

$$M_o = C_b \pi \sqrt{\frac{EI_y GJ}{l_{ey}}}\sqrt{1 + \frac{\pi^2 EI_w}{GJl_{ez}^2}} \quad (5.16)$$

This is the same as Eq. (5.8) except that:

(i) C_b in the numerator replaces C_{TF} in the denominator, and

(ii) the unbraced length (l) is replaced by l_{ey} for the unbraced flexural length about the *y*-axis, and l_{ez} for the unbraced torsional length, as appropriate.

Hence Eq. (3.3.3.2(8)) in Clause 3.3.3.2.1(a)(i) of AS/NZS 4600 is a more general version of Eq. (5.8) with the C_b factor replacing the reciprocal of the C_{TF} factor. The C_b factor is more general than the C_{TF} factor given by Eq. (5.9) since it allows for a moment distribution which is not simply linear as shown in Fig. 5.2(a). It will be discussed in more detail in Section 5.2.2 following.

In Clause 3.3.3.2.1(b) of AS/NZS 4600, a specific equation (3.3.3.2(17)) for the elastic buckling moment of a point-symmetric Z-section is given as

$$M_o = \frac{\pi^2 E C_b d I_{yc}}{2l^2} \tag{5.17}$$

where I_{yc} is the second moment of area of the compression portion of the section about the centroidal axis of the full section parallel to the web, using the full unreduced section. This equation can be derived from Eq. (5.11) by putting $I_w=I_y d^2/4$, $I_{yc}=I_y/2$, $C_b=1/C_{TF}$ and including an additional factor ½ to allow for the fact that a Z-section has an inclined principal axis whereas I_{yc} is computed about the centroidal axis parallel to the web. The resulting simplified formula has been used successfully in the USA for many years for the lateral buckling of Z-sections.

5.2.2 Continuous Beams and Braced Simply Supported Beams

In practice, beams are not usually subjected to uniform moment or a linear moment distribution, and are not always restrained by simple supports. Hence if an accurate analysis of flexural-torsional buckling is to be performed, the following effects should be included:

(a) Type of beam support including simply supported, continuous and cantilevered.

(b) Loading position including top flange, shear centre and bottom flange.

(c) Positioning and type of braces (commonly called bridging for purlins).

(d) Restraint provided by sheeting including the membrane, shear and flexural stiffnesses.

A method of finite element analysis of the flexural-torsional buckling of continuously restrained beams and beam-columns has been described in Ref. 5.4 and was applied to the buckling of simply supported purlins with diaphragm restraints in Ref. 5.5 and continuous purlins in Ref. 5.3. The element used in these references is shown in Fig. 5.3(a) and shown subjected to loading in Fig. 5.3(b). The loading allows for a uniformly distributed load located a distance ($\bar{a}$) below the shear centre.

A computer program PURLIN has been developed at the University of Sydney to perform a flexural-torsional buckling analysis of beam-columns and plane frames using the theory described in Refs 5.2 and 5.4.

The method has been applied to the buckling of simply supported beams subjected to uniformly distributed loads as shown in Fig. 5.2(b) to determine suitable C_b factors for use in AS/NZS 4600 Clause 3.3.3.2. The loading was located at the tension flange, shear centre axis and compression flange. Three different bracing configurations were used ranging from no intermediate bracing, through central bracing to third point bracing. Each brace, including the end supports, was assumed to prevent both lateral and torsional deformation. The element subdivisions used in the analysis are shown in Fig. 5.4 for both central and third point bracing. The resulting lateral deflections of the shear centre in the buckling mode are also shown in Fig. 5.4.

Design of Cold-Formed Steel Structures

(To Australian/New Zealand Standard AS/NZS 4600:2005)

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