AUSTRALIAN STEEL INSTITUTE

AS 4100 DS05

Steel Structures – Second-order analysis of compression members

Written by: N.S. Trahair
Date: February 1993

Checked by: R.Q. Bridge
Date: February 1993

SECOND-ORDER ANALYSIS OF COMPRESSION MEMBERS

INTRODUCTION

The AS 4100 Steel Structures standard generally allows the effects of second-order moments to be approximated by multiplying the first-order elastic moments (Ref. 1) by an amplification factor (δ), except when δ is greater than 1.4. Most practical structures have amplification factors less than 1.4.

However, some members with large compression loads and small transverse loads may have amplification factors greater than 1.4. Examples include horizontal chords of trusses whose self-weights are considered to be important. If the amplification factor is greater than 1.4, then AS 4100 requires a second-order elastic analysis to be used to determine the design bending moments.

Close approximations for the results of the second-order elastic analyses which can be used in design are presented in the following sections. They can be used for members that are simply supported, cantilevered, build-in at one or both ends, or continuous over two spans, and loaded with either uniformly distributed load, central or end concentrated load, or end moments.

Second-order elastic analysis

Second-order elastic analyses of compression members need to include the second-order moments $N\nu$ exerted by the compression forces N as a result of the deflections ν. It is conventional to ignore other small effects such as those due to shear straining, finite joint size, large deflections and rotations, and member length changes resulting from elastic shortening and bowing.

Second-order elastic analyses may be carried out by including the second-order moments $N\nu$ in the classical differential equations for the first-order engineering theory of elastic bending. A worked example of a member with unequal end moments, M and $\beta_m M$ is provided in Section 7.8.1 of Ref. 2. In this case, the amplification factor δ, which is the ratio of the maximum second-order moment M to the maximum first-order moment M_m, is given by

$$\delta = 1.0 \qquad \ldots(1)$$

while $\beta_m \geq -\cos\pi\sqrt{(N/N_{om})}$ ...(2)

in which N_{om} is the member elastic buckling load $\pi^2 EI/l^2$, and by

$$\delta = \sqrt{\{1+[\beta_m \operatorname{cosec}\pi\sqrt{(N/N_{om})} + \cot\pi\sqrt{(N/N_{om})}]^2\}} \qquad \ldots(3)$$

otherwise.

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Amplification factors

Second-order elastic analyses have been made of the compression members shown in Tables 1 and 2 by using the differential equation method demonstrated in Section 7.8.1 of Ref. 2. Close approximations for the solutions for the maximum second-order elastic moments M can be obtained from

$$M = \delta M_m \qquad \ldots(4)$$

in which

$$\delta = \frac{\gamma_m(1-\gamma_s N/N_{om})}{(1-\gamma_n N/N_{om})} \geq 1 \qquad \ldots(5)$$

and

$$N_{om} = \pi^2 EI/k^2 l^2 \qquad \ldots(6)$$

In these equations, M_m is the maximum first-order moment, γ_m, γ_n and γ_s are constants obtained by curve fitting the results of the analyses, and k is the member effective length factor. Values of k, γ_m, γ_n and γ_s, and M_m are given in Tables 1 and 2.

Worked example

The two-span continuous column shown in Figure 1 is a 200UC52.2 of Grade 250 steel. It has a design compression load of N^* = 400 kN and a design uniformly distributed load of w^* = 0.64 kN/m causing bending about the minor axis.

Using Equation 6,

$N_{om} = \pi^2$ x 2.0E5 x 17.7E6 / (1.0 x 7000)2N

= 713.0 kN

Using Clause 4.4.2.2 conservatively with

β_m = -1.0 so that c_m = 1.0, then

δ_b = 1.0 / (1 – 400/713.0) = 2.28 > 1.4

And so it seems that a second-order analysis must be used.

Using the second-order analysis approximation of Equation 5 and 6 with the values of γ_m = 1.0, γ_n = 0.49, and γ_s = 0.18 obtained from Table 1, then

$$\delta = \frac{1.0 \times (0.18 \times 400/713.0)}{(1-0.49 \times 400/713.0)} = 1.24$$

The first-order elastic moment is

$$M_m^* = w^* l^2 / 8 = 0.64 \times 7^2 / 8 = 3.92 \text{ kNm}$$

which can be used in Equation 4 to obtain the second-order elastic moment as

$$M^* = 1.24 \times 3.92 = 4.86 \text{ kNm}$$

Note that if the approximation of β_m = 0.2 obtained directly from Table 4.4.2.2 had been used in Clause 4.4.2.2 then

δ_b = (0.6 – 0.4 x 0.2) / (1 – 400/713.0) = 1.18 < 1.4

so that a second-order analysis need not have been used.

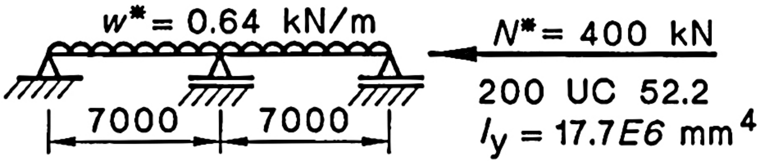


FIGURE 1 TWO-SPAN CONTINUOUS COLUMN

Discussion

In many cases, it is conservative to use Clause 4.4.2.2 of AS 4100 to amplify the first-order elastic moments, even when the amplification factor is greater than 1.4. However, there may be some examples when this is not conservative, as in the worked example above. In any case, it is expected that the use of Clause 4.4.2.2 to amplify the first-order moments will be sufficiently accurate for preliminary design purposes. This is illustrated in the example below.

A 9.0 m long horizontal roof strut in a portal frame roof truss is a 114 x 5.4 CHS of Grade C250 steel (Ref. 3). It has a design compression load of N* = 36.6 kN and a design uniformly distributed load of w^* = 0.178 kN/m.

Using Equation 6,

$N_{om} = \pi^2 \times 2.0E5 \times 2.75E6 / (1.0 \times 9000)^2 N$

$= 67.0$ kN

Using the second-order analysis approximation of Equations 5 and 6 with the values of γ_m = 1.0, γ_n = 1.0, and γ_s = -0.03 obtained from Table 1, then

$$\delta = \frac{1.0 \times (1 + 0.03 \times 36.6/67.0)}{(1 - 1.0 \times 36.6/67.0)} = 2.24$$

Using Table 4.4.2.2, β_m = -1.0, so that c_m = 1.0, and so

$$\delta_b = 1.0 / (1 - 36.6/67.0) = 2.20$$

which is very close to the accurate value of 2.24.

REFERENCES

1 Trahair, N.S., (1992), 'Moment Amplification of First-Order Analysis', *Limit States Data Sheet AS 4100 DS03*, Australian Institute of Steel Construction and Standards Australia, Sydney

2 Trahair N.S., and Bradford M.A., (1991), *The Behaviour and Design of Steel Structures*, Revised Second Edition, Chapman and Hall, London.

3 Woolcock, S.T., Kitipornchai, S., and Bradford, M.A., (1993), *Limit State Design of Portal Frame Buildings*, 2nd Edition, Australian Institute of Steel Construction, Sydney.

TABLE 1
UNIFORMLY DISTRIBUTED LOADS

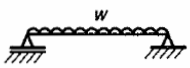
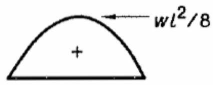
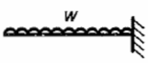
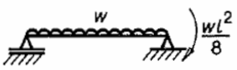
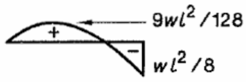
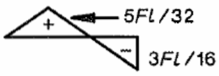
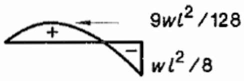
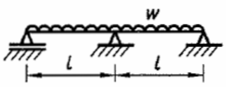
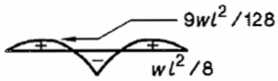
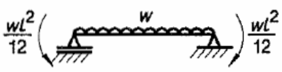
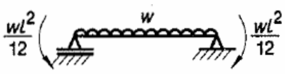
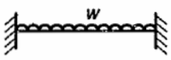
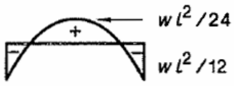
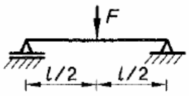
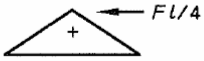
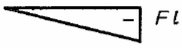
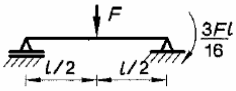
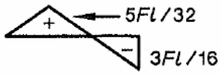
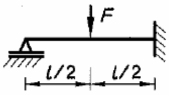
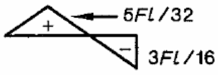
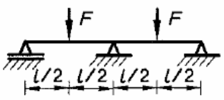
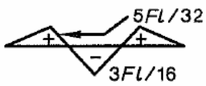
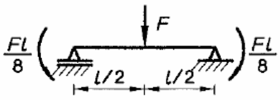
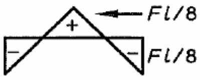
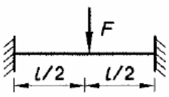
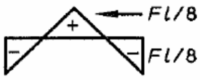
Member	First-order moment	k	γ_m	γ_n	γ_s	γ_m
		1.0	1.0	1.0	−0.03	$\frac{wl^2}{8}$
		2.0	1.0	1.0	0.4	$\frac{wl^2}{2}$
		1.0	$\frac{9}{16}$	1.0	0.29	$\frac{wl^2}{8}$
		0.7	1.0	1.0	0.36	$\frac{wl^2}{8}$
		1.0	1.0	0.49	0.18	$\frac{wl^2}{8}$
		1.0	0.5	1.0	0.4	$\frac{wl^2}{12}$
		0.5	1.0	1.0	0.37	$\frac{wl^2}{12}$

TABLE 2
CENTRAL CONCENTRATED LOADS

Member	First-order moment	k	γ_m	γ_n	γ_s	M_m
		1.0	1.0	1.0	0.18	$\frac{Fl}{4}$
		2.0	1.0	1.0	0.19	Fl
		1.0	$\frac{5}{6}$	1.0	0.45	$\frac{3Fl}{16}$
		0.7	1.0	1.0	0.28	$\frac{3Fl}{16}$
		1.0	1.0	0.49	0.14	$\frac{3Fl}{16}$
		1.0	1.0	1.0	0.62	$\frac{Fl}{8}$
		0.5	1.0	1.0	0.18	$\frac{Fl}{8}$