ANALYSIS-BASED 2D DESIGN OF STEEL STORAGE RACKS

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ABSTRACT

The report presents a study of the capacities of steel rack frames based on linear analysis (LA), geometric nonlinear analysis (GNA) and geometric and material nonlinear analysis (GMNIA). In the case of linear and geometric nonlinear analyses, the design is carried out to the Australian cold-formed steel structures AS/NZS4600. The study includes braced, unbraced and semi-braced frames, and compact and non-compact cross-sections. The report shows axial force and bending moment paths for geometric and geometric and material nonlinear analyses, and explains the differences observed in the design capacities obtained using the different types of analysis on the basis of these paths. The report provides evidence to support the use of advanced geometric and material nonlinear analysis for the direct design of steel rack frames without the need for checking section or member capacities to a structural design standard.

KEYWORDS

Steel Storage Racks, Australian Standard, Advanced Analysis, Design.

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INTRODUCTION

Current specifications for steel structures [1-3] allow the design to be based on “advanced” geometric and material nonlinear analysis. In the Australian Standard AS4100 [1], inelastic second order effects may be determined by advanced analysis, requiring only the cross-section and connection capacities to be determined according to the Standard. In Eurocode3, Part 1.1 [3], and the American Specification (AISC-LRFD) [2], a similar approach is permitted except that the member interaction strength equations for combined actions are required to be used even when the internal stress resultants are determined from advanced geometric and material nonlinear analyses. In all cases, the cross-section must be compact and members must be fully braced against torsion and lateral buckling.

When first published in 1990, the Australian Standard (AS4100) included provisions for geometric nonlinear elastic analysis (often termed “2nd order” analysis) as well as geometric and material nonlinear analysis. Commercial software featuring 2nd order analysis was developed soon after and has been employed increasingly in design offices as the basis for structural design over the last 20 years, thus obviating the need for amplification of moments determined by linear-elastic small-displacement (“1st order”) analysis. It is now common practice in Australia to use geometric nonlinear analysis for design. However, mainstream commercial structural analysis software packages have not included geometric and material nonlinear analysis, partly because of (i) the greater complexity of specifying material properties in such analyses and (ii) the requirement to include geometric imperfections and residual stresses, which are generally not defined in structural design standards, and partly because national design standards still require the section or member capacity to be checked, thus effectively negating efficiencies to be gained by using geometric and material nonlinear analysis for direct structural design. In effect, the only benefit to be gained from employing geometric and material nonlinear analysis over geometric nonlinear analysis is that the obtained internal stress resultants are more rationally based. Interestingly however, the recently released Version 2.4.1 of the widely industry-used Australian software package Strand7 [4] includes the capability to analyse structural frames by geometric and material nonlinear analysis as per the method described in Clarke and al. [5]. This capability will stimulate design engineers’ interest in using this type of analysis in design, particularly if (i) the design is allowed to be based directly on the nonlinear analysis without an imposed recourse to interaction equations in national standards and (ii) the current scope of geometric and material nonlinear analyses is broadened to include slender cross-sections and non-fully braced members failing by flexure and torsion.

While geometric and material nonlinear analysis has not yet been generally embraced in design practice, research institutions have used advanced analysis finite element packages like Abaqus, Ansys, Nastran, Marc and Lusas for several decades and it is now well established that the behaviour of structural steel frames can be very accurately predicted using advanced analysis, provided all features affecting the behaviour are included in the analysis, notably geometric and material nonlinearities as well as imperfections. The literature features a wealth of articles demonstrating that the structural behaviour of members and systems subject to complex buckling modes, (e.g. local, distortional, flexural and flexural-torsional modes) and/or complex material characteristics can be modelled accurately using advanced finite element software.

In view of these advances, when Standards Australia initiated a review of the Australian Standard for Steel Storage Racks, AS4084:1993 [6], the Standards committee charged with the review decided to include provisions for designing steel storage racks by advanced analysis. This required an articulation of the features required to be modelled in using geometric and material nonlinear analysis, notably guidance on which imperfections to include and their magnitudes. The draft Standard [7] acknowledges that the analysis may be based on shell element analysis in order to appropriately model the effects of local and distortional buckling and includes provisions for this type of analysis. It also allows for flexural-torsion buckling of the structural members. The main features of the advances made in the new draft Standard [7] are detailed in [8].

The main objective of this report is to investigate the consistency of using different types of analysis as basis for structural design. Hence, case studies are presented for the design of steel storage racks based on linear-elastic, geometric nonlinear and geometric and material nonlinear analyses. Three different bracing configurations and two distinct cross-sections are considered, including a non-compact section which is subject to distortion of the cross-section in the ultimate limit state. Failure modes involving flexural and flexural-torsional buckling are investigated. To reduce the number of parameters, perforations are not included in this study and the frames are assumed to be braced in one direction so as to limit displacements to occur in a single plane, with or without torsion of uprights.

METHODS AND SCOPE OF ANALYSIS

The draft Standard [7] includes provisions for design to be carried out on the basis of the following types of analysis:

- LA — Linear ("1st order") Analysis assuming elastic material and small displacements.
- GNA — Geometric Nonlinear ("2nd order") Analysis assuming large displacements.
- LBA — Linear Buckling Analysis assuming linear fundamental path.
- GMNIA — Geometric and Material Nonlinear ("advanced") Analysis with Imperfections assuming large displacements and inelastic material properties.

A linear buckling analysis (LBA) may be required when using LA analysis to determine moment amplification factors. The draft Standard distinguishes between two types of GMNIA analysis, namely analyses of frames with compact cross-section (GMNIAc), which may be premised on beam elements, and analyses of frames with non-compact cross-section (GMNIAs) which require shell or plate element discretisation to capture the effects of local and distortional buckling deformations.

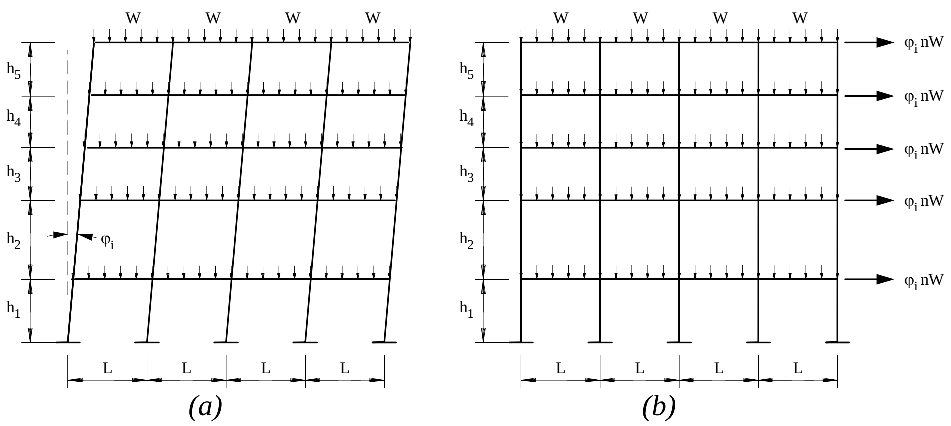


Figure 1: Frame imperfection, (n is the number of bays), (a) Typical unbraced rack showing initial out-of plumb (φ_i) (b) Equivalent loading system for the unbraced rack

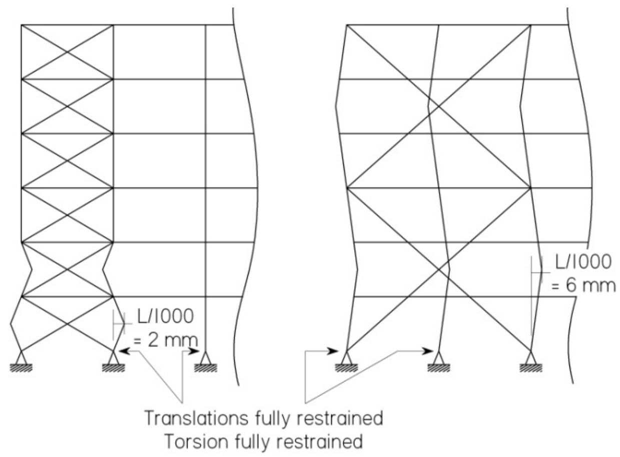


Figure 2: Member imperfection

Particular attention is paid to the modeling of geometric imperfections in the draft Standard. Irrespective of the type of analysis, out-of-plumb ("frame") imperfections are modeled by means of equivalent notional horizontal forces, see Fig. 1. In LA, GNA and LBA analyses, this is the only type of imperfection modeled. It is implicit that the effects of out-of-straightness of members between connection points ("member" imperfections) and out-of-flatness of component plates of cross-sections ("section" imperfections) are considered by using member (column and beam) strength curves and plate/section (e.g. effective width) strength curves of structural design standards, respectively. In using GMNIAc and GMNIAs analyses, member imperfections must be modeled which can be achieved by (i) superimposing a scaled buckling mode of an equivalent frame with all beam levels restrained horizontally, (ii) reducing the flexural rigidity to 80% of its elastic value or (iii) off-setting nodes of uprights by an amplitude of $L/1000$, as shown in Fig. 2. In using GMNIAs analysis, local and distortional geometric imperfections are also required to be modeled (Fig. 3), e.g. by superimposing scaled local and distortional buckling modes onto the flat section geometry.

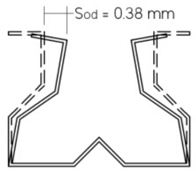


Figure 3: Section imperfection

The draft Standard requires residual stresses to be modeled in GMNIA analyses when significant. The sections considered in this study are assumed to be cold-formed and to have negligible levels of residual stress. Accordingly, residual stresses are not considered.

The draft Standard allows LBA and GNA analyses to be carried out considering or not considering torsion. In this study, torsion has not been included in LA, LBA and GNA analyses and consequently, the amplification of bending moments in GNA analyses is caused solely by instability related to flexural displacements. As mentioned in the Introduction, displacements are assumed to occur solely in the down-aisle direction.

The GMNIAc analyses are conducted for the distinct cases of torsion and no torsion of the uprights. The GMNIAs analysis based on shell element discretisation allows torsional deformations to develop. However, the cross-aisle displacement of the uprights is fully restrained, as shown in Fig. 4.

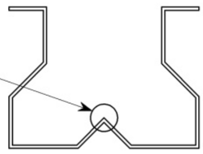


Figure 4: Cross-aisle restraint of uprights

CASE STUDIES

STEEL STORAGE FRAMES

The rack frames considered in this study have the same common overall geometry, consisting of five 3.4 m wide bays and six beam levels equally spaced 2 m apart, as shown in Fig. 5. The frames may be unbraced, fully braced or semi-braced, as also shown in Fig. 5. In the semi-braced configuration, the third and upper beam levels are essentially restrained horizontally. Two cross-sections are considered for the uprights, namely a 100×100×6 mm box section and a rear-flange stiffened rack section with a web width of 110 mm and a thickness of 1.5 mm, referred to as RF11015, as shown in Fig. 5. In all analyses, the pallet beams and diagonal bracing are assumed to be compact 60×60×4 mm SHS and compact 30×2 mm CHS, respectively.

Table 1: Geometric properties of upright cross-sections

Property	Section	
	100×100×6 SHS	**RF11015**
A (mm^2)	2256	508
I_y (mm^4)	3.33×10^6	4.46×10^5
J (mm^4)	5.00×10^6	381
I_w (mm^6)	-	1.30×10^9
x_0 (mm)	0	0
y_0 (mm)	0	-67.5

The section constants of the two upright cross-sections are shown in Table 1, where A is the area, I_y the second moment of area about the y-axis, which is the symmetry axis aligned with the cross-aisle direction, J the torsion constant, I_w the warping constant and (x_0,y_0) the shear centre coordinates.

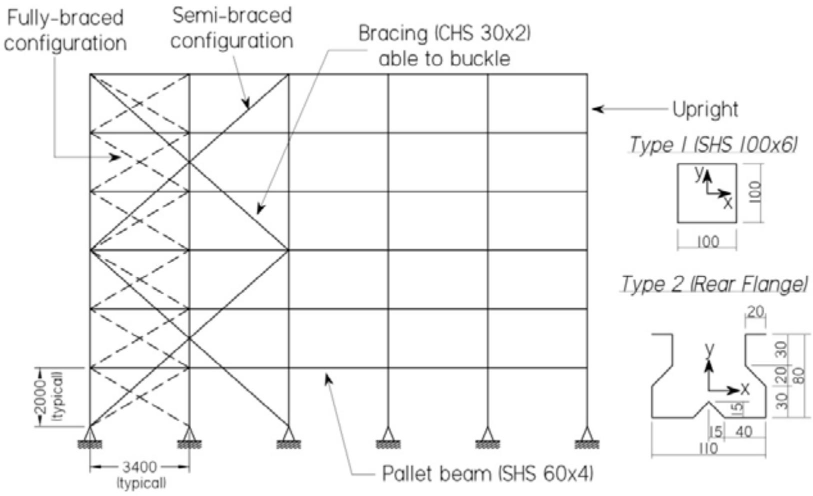


Figure 5: Rack configurations and upright cross-sections

At the base, the uprights are assumed to be simply supported against flexure in the down-aisle direction while prevented against warping. The frame is assumed to be braced against displacements in the transverse (cross-aisle) direction. In the LA, GNA and GMNIAc analyses, the connections between the uprights and pallet beams are assumed to be rigid. The connection between uprights and pallet beams in the GMNIAs analyses is explained in the subsequent section.

All uprights are assumed to have a yield stress (f_y) of 450 MPa, while all pallet beams and diagonal bracing members are assumed to remain elastic. The engineering stress-strain curve for the uprights is assumed to be linear perfectly-plastic in the GMNIA analyses, thus ignoring the effects of strain hardening.

Analysis models and results

The finite element analyses were carried out using the commercial packages Strand7 [4] and Abaqus [9], as summarised in Table 2.

Table 2: Analysis types and software

Analysis	Torsion of uprights	Software
LA, LBA, GNA	No	Strand7
GMNIAc	No	Strand7
GMNIAc	Yes	Abaqus
GMNIAs	Yes	Strand7

In the GMNIAc analyses which consider torsion of the uprights, both uniform (St Venant) torsion and warping torsion are included. The failure modes in these analyses are dominated by flexural-torsional buckling of the uprights.

The LA, LBA, GNA and GMNIAc (no torsion) analyses use the general purpose beam element of the Strand7 library, while the GMNIAc analysis (torsion) use the beam element B32OS of the Abaqus library for the uprights and beam element B33 for the remaining members. The GMNIAs analyses are carried out using the general purpose shell element of the Strand7 library. In the GMNIAs analyses, the uprights are supported and connected to pallet beams using rigid beam elements, as shown in Fig. 6. The rigid links restrain warping at the base while allowing flexural rotations and applying a concentric reaction force. The rigid links also restrain warping of the web of the uprights but not the flanges at the pallet beam connection points. In effect, the pallet beam connections offer very minor warping restraints to the uprights, while producing a flexurally rigid connection.

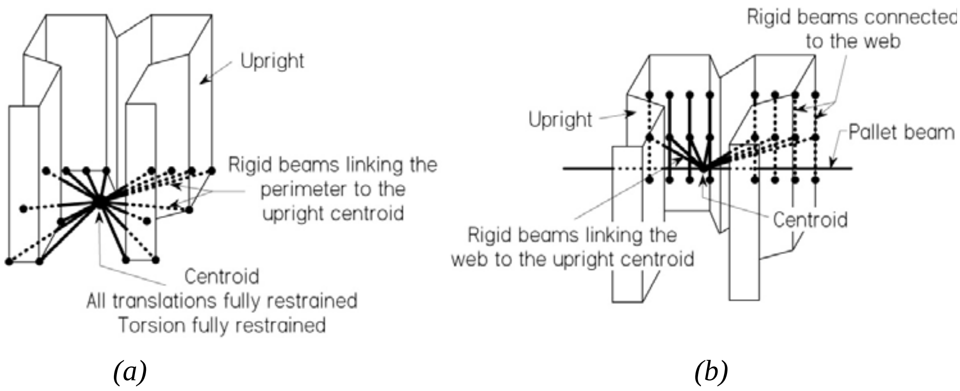


Figure 6: GMNIAs modeling of base plate support and upright to pallet beam connection, (a) FE model of support at the base and top of upright, (b) FE model of upright to pallet beam connection

The rack is assumed to be subject to a vertical force (*P*) at each upright to pallet beam connection point (fully loaded). According to the draft Standard [7], notional horizontal forces of $\phi_s V$ are applied at each beam level, where ϕ_s is the out-of-plumb and *V* is the total vertical load applied as the particular level, i.e. 6*P* for the present study. The out-of-plumb depends on the tolerance grade, as per Table 3, and the type of structural analysis. For all ultimate limit states analyses, including GMNIA analyses, a minimum value of 1/500 is required, while for GNA analysis, a minimum value of ϕ_s of 1/333 is required.

The analyses reported in this report are obtained using out-of-plumb values of 1/333 for LA and GNA analyses, and values of 1/333, 1/500 and 1/1000 for GMNIA analyses. Ordinarily, an out-of-plumb value of 1/500 would be used for GMNIA analysis. The additional values of 1/333 and 1/1000 are included in this study to investigate the sensitivity of the frame capacity to out-of-plumb.

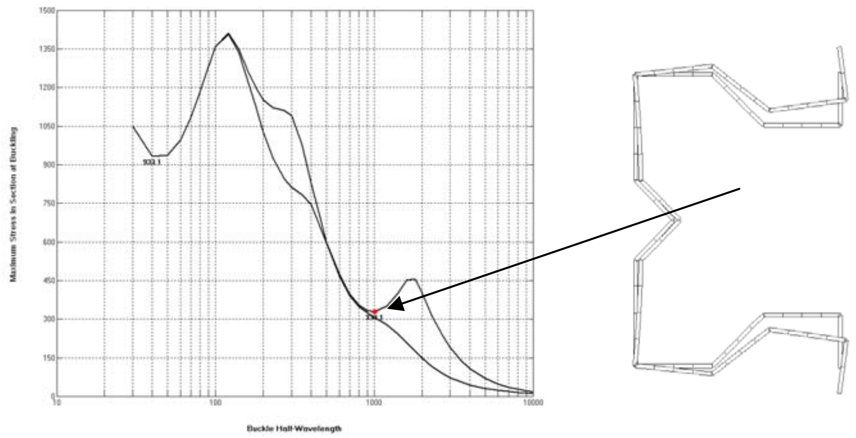
Member imperfections are included in the GMNIA analyses of the braced and semi-braced frames as per the draft Standard. It is not considered necessary to include member imperfection in the GMNIA analyses of the unbraced frames as the P-δ (member) moment amplification is negligible compared to the P-Δ (frame) moment amplification for these frames. According to the draft Standard, the magnitude of the member imperfections is taken as *L*/1000 where *L* is the vertical distance between the bracing points, i.e. *L*=2 m for the fully braced frame and *L*=6 m for the semi-braced frame, as shown in Fig. 2.

Table 3: Out-of-plumb (ϕs) as per draft Standard

Tolerance grade	Type of unit load handling equipment	Out-of-plumb (ϕ_s)
I	Manually operated equipment guided by operator	1/500
II	Manually operated equipment guided by electrical or mechanical devices	1/750
III	Fully automatic operated equipment guided by electrical or mechanical devices	1/1000

The local and distortional buckling modes and buckling stresses of the RF10015 section are determined using Thinwall [10]. The graph of buckling stresses *vs* half-wavelength for pure compression is shown in Fig. 7a for the first two buckling modes. Figure 7b shows the distortional buckling mode, obtained as the second mode at a half-wavelength of 1000 mm. The distortional buckling stress for pure compression is obtained as f_{od}=330 MPa. The critical local buckling stress for uniform compression is determined as f_{ol}=933 MPa. This is substantially higher than the yield stress and so local buckling will not occur before reaching the ultimate capacity. Consequently, local buckling imperfections are not included in the GMNIAs analysis.
According to the draft standard, the magnitude of the imperfection in the shape of the distortional buckling mode is determined as,

$$S_{od} = 0.3t\sqrt{\frac{f_y}{f_{od}}} \quad (1)$$



(a) (b)

Figure 7: Buckling stress vs half-wavelength and distortional buckling mode, (a) Buckling stress vs half-wavelength, (b) Distortional buckling mode

The distortional imperfection is incorporated in the GMNIAs analysis by linearly flaring the flanges between the ends and the centres of the uprights, as exemplified in Fig. 8.

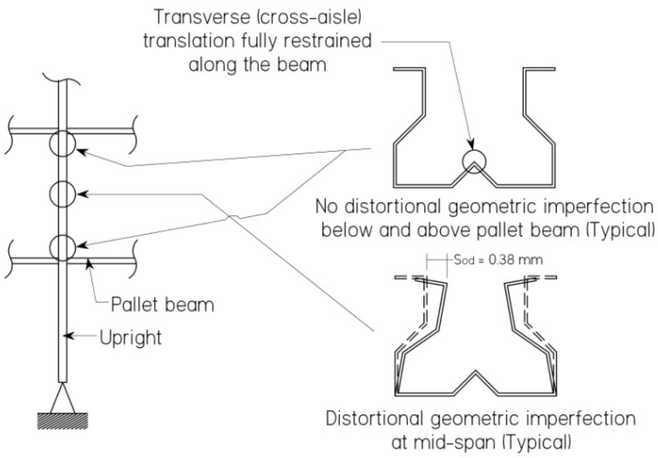


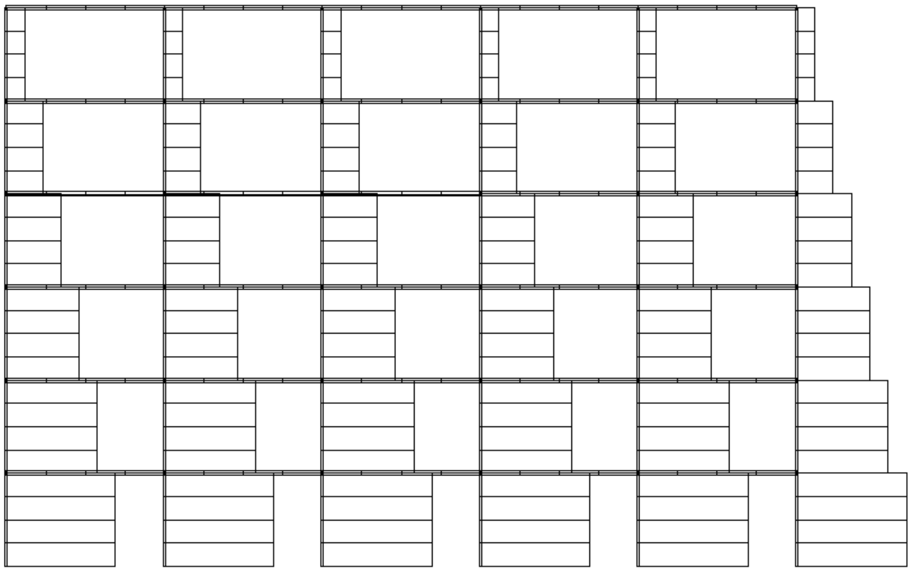
Figure 8: Modeling of the distortional imperfection in GMNIAs analysis

The buckling loads (P_c) of the frames are determined for all bracing configurations using an LBA analysis, as summarised in Table 4. The corresponding buckling modes are shown in appendix 1 of the present report. The buckling load (P_{cb}) of the equivalent fully horizontally restrained frame is also determined by preventing horizontal displacements of each level of the frame. The buckling load of the laterally restrained frame is close to that of the fully braced frame.

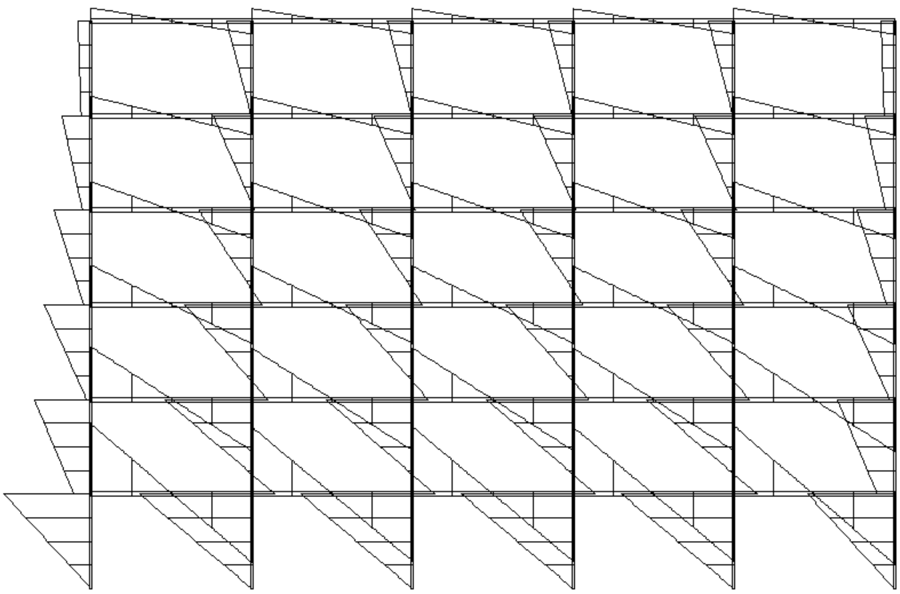
Table 4: Frame buckling loads (Pc) in kN.

Bracing arrangement	**Section**	
	100×100×6 SHS	**RF11015**
P_c (kN) for unbraced rack	20.9	11.0
P_c (kN) for fully braced rack	337	99.6
P_c (kN) for semi-braced rack	64.5	22.7
Laterally restrained frame (P_{cb})	342	95.4

The LA and GNA analyses produce axial force (N) and bending moment (M) distributions in the frames for given values of applied vertical and horizontal forces. The axial force and bending moment distributions are shown in Figs 9a and 9b for the unbraced rack with 100×100×6 mm SHS uprights, as determined from an LA analysis. The axial force attains its maximum value between the support and the first beam level, and decreases gradually with increasing beam level. The maximum bending moment is generally found between the floor and the first beam level, and so the critical (N,M)-combinations are found for the uprights between the floor and the first beam levels. Similar axial force and bending moment distributions are shown for the fully braced and semi-braced frames in appendix 2 of the present report.



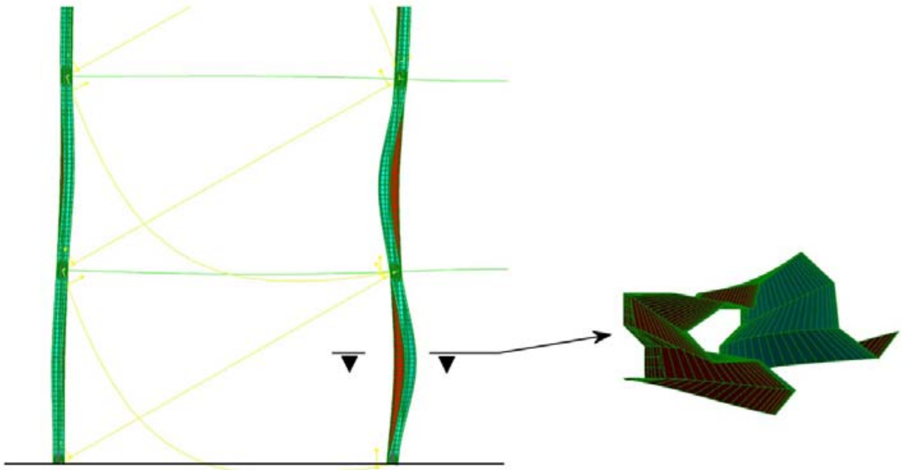
(a) Axial force distribution



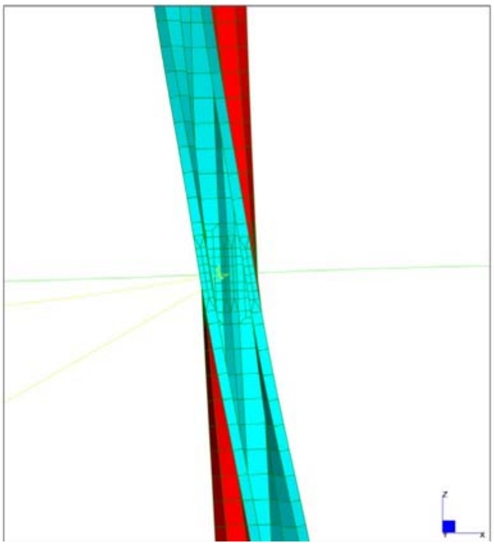
(b) Bending moment distribution

Figure 9: Axial force and bending moment distributions in unbraced rack as determined by LA analysis

The frames fail by inelastic flexural buckling of the uprights between the floor and the first beam level in all GMNIA analyses not subject to torsion. When torsion is considered, the overall failure mode is by flexural-torsional buckling of the uprights between the floor and the first beam level and the uprights between the first and second beam level. In the GMNIAs analysis, failure is also associated with distortional buckling, as shown in Figs 10a and 10b for the fully braced frame. The ultimate loads (P_u) determined from GMNIA analyses are summarised in Discussion.



(a) Failure mode of fully braced frame; close-up of first- and second-most uprights near the base. Torsion and distortion of the upright are evident.



(b) Torsion and warping of critical upright near pallet beam connection point

Figure 10: Failure mode of fully braced frame as obtained from GMNIAs analysis

Basis of design

LA and GNA analyses.

The ultimate capacity of the frame (P_u) is determined by calculating the axial (N_c) and flexural (M_b) capacities of the uprights using the Australian Standard for Cold-formed Structures AS/NZS4600:2005 [11] and requiring the interaction equation be satisfied,

$$\frac{N^*}{\varphi_c N_c} + \frac{M^*}{\varphi_b M_b} = 1 \tag{2}$$

In eq. (2), N^* and M^* are the axial force and maximum amplified bending moment in the upright, which are functions of the design load (P_u), and φ_c=0.85 and φ_b=0.9 are resistance factors for compression and bending respectively. For LA analysis, as per the draft Standard, the amplified bending moment is determined as

$$M^* = \frac{M^*_{LA}}{\alpha} \tag{3}$$

where M^*_{LA} is the 1st order moment determined from the LA analysis, and α is the moment amplification factor, calculated as

$$\alpha = \frac{N_e}{N_e - N^*} \tag{4}$$

where N_e is the buckling load of the upright, as determined from an LBA analysis. It follows that the amplification factor may be calculated as,

$$\alpha = \frac{P_c}{P_c - P_u} \tag{5}$$

where P_c is the buckling load of the frame, see Table 4, and P_u is the ultimate design load of the frame which is the object of the calculation.

For LA analysis, eqs (2-5) lead to a quadratic equation in P_u while for GNA, increasing values of (N^*, M^*) are substituted into the left-hand side of eq. (2) until the equation is satisfied, thus determining the ultimate value of load (P_u).

The axial (N_c) and flexural (M_b) capacities of the uprights are obtained according to the Direct Strength Method included in Section 7 of AS/NZS4600 [12], and so account for local and distortional buckling. The axial capacity (N_c) is based on the overall flexural buckling load when torsion is not considered, and the flexural-torsional buckling load when torsion is considered. Because cross-aisle displacements are restrained, flexural-torsional buckling will not occur as a result of bending and hence, the moment capacity for bending about the y-axis of symmetry (M_b) is based on the yield moment. The moment capacity is reduced by distortional buckling for the non-compact cross-section.

In determining the axial capacity (N_c), the effective length (L_{ey}) for bending about the y-axis is calculated from the buckling load (P_{cb}, see Table 4) of the frame with all beam levels restrained horizontally, as per the draft Standard. As shown in appendix 1 of the present report, the effective length for bending about the y-axis is generally about 90% of the member length (L=2 m). In view of the warping restraint at the base and the small warping restraint at the connection points between uprights and pallet beams, the torsional effective length (L_{ez}) is determined as 0.7L for the lengths of upright between the floor and the first beam level, and as 0.9L for the uprights between the first and second beam levels. Because of the larger torsional effective length for the uprights between the first and second beam levels, these uprights prove critical in determining the beam-column capacity (i.e. satisfying eq. (2)) in the designs where torsion is considered.

GMNIA analyses.

The analysis provides the maximum load (P_{max}) which can be applied to the frame. Depending on the elements used in the analysis, torsion and cross-sectional distortion are accounted for. According to the draft Standard, the ultimate capacity (P_u) is determined as,

$$P_u = \varphi_s P_{\max} \tag{6}$$

where φ_s=0.9 is the system resistance factor.

DISCUSSION

The ultimate design capacities obtained on the basis of LA, GNA and GMNIA analyses are summarised in Table 5 for the various combinations of bracing arrangement, upright cross-section, and allowance for torsion and cross-sectional instability. The six columns on the right provide the ratios between the capacities based on LA and GNA analyses and the strength obtained using GMNIA analysis, where GMNIA implies GMNIAc when the cross-section is assumed compact and GMNIAs when the cross-section is assumed non-compact. It can be seen that the design capacities predicted on the basis of LA and GNA analyses are close for all bracing configurations and that, on an average basis, the difference between LA and GNA analysis-based capacities and GMNIA-based capacities is ±1% for ϕ_s=1/500. Calculation of the ultimate design capacities are detailed in appendix 1.

Table 5: Design capacities (Pu)

Upright cross-section	Bracing	Compact/ non-compact	Torsion of uprights in GMNIA	Design capacity (P_u) in kN LA	GNA	GMNIA ϕ_s=1/1000	GMNIA ϕ_s=1/500	GMNIA ϕ_s=1/333	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/1000	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/500	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/333	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/1000	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/500	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/333
SHS	unbraced	compact	no	20.2	20.0	18.7	18.5	18.2	1.08	1.09	1.11	1.07	1.08	1.10
	braced	compact	no	113.7	113.6	136.4	134.7	131.6	0.83	0.84	0.86	0.83	0.84	0.86
	semi-brac	compact	no	62.8	61.7	53.4	53.1	52.8	1.18	1.18	1.19	1.15	1.16	1.17
RF11015	unbraced	compact	no	9.96	9.84	9.58	9.32	9.09	1.04	1.07	1.10	1.03	1.06	1.08
	braced	compact	no	27.3	27.3	31.5	31.1	30.6	0.87	0.88	0.89	0.87	0.88	0.89
	semi-brac	compact	no	21.1	19.8	17.9	17.7	17.6	1.18	1.19	1.20	1.11	1.12	1.13
RF11015	unbraced	compact	yes	9.59	9.49	9.51	9.26	9.00	1.01	1.04	1.07	1.00	1.02	1.05
	braced	compact	yes	15.3	15.3	19.3	19.1	19.0	0.79	0.80	0.81	0.79	0.80	0.81
	semi-brac	compact	yes	15.0	15.0	16.8	16.7	16.6	0.89	0.90	0.90	0.89	0.90	0.90
RF11015	unbraced	non-comp	yes	9.30	9.16	7.88	7.48	7.29	1.18	1.24	1.28	1.16	1.22	1.26
	braced	non-comp	yes	15.3	15.3	18.4	18.2	18.0	0.83	0.84	0.85	0.83	0.84	0.85
	semi-brac	non-comp	yes	14.9	14.9	14.9	14.8	14.8	1.00	1.01	1.01	1.00	1.01	1.01
Average									**0.99**	**1.01**	**1.02**	**0.98**	**0.99**	**1.01**

However, the capacity ratios are clearly biased towards the bracing configuration. This is brought out in Table 6 which separately lists the averages of the capacity ratios for unbraced, fully braced and semi-braced racks. Evidently, GMNIA analysis-based design capacities are consistently conservative for LA and GNA analysis-based capacities for unbraced frames and consistently optimistic for LA and GNA analysis-based capacities for fully braced frames, irrespective of the out-of-plumb value (ϕ_s). For semi-braced frames, GMNIA analysis-based design capacities are conservative compared to LA and GNA analysis-based capacities when the uprights fail by flexure but may be optimistic when the uprights fail by flexural-torsional buckling.

The capacity of the braced rack frames is largely governed by the axial capacity of the uprights, i.e. the $N^*/\varphi_c N_c$-term dominates the left-hand side of eq. (2). To investigate the cause of the optimism of the GMNIA analysis-based design capacities for braced frames, a single concentrically loaded box-section upright with an imperfection of L/1000 at the centre is analysed using GMNIAc analysis. The column length (L) is taken equal to the effective column length based on an LBA analysis, i.e. L=1.83m, producing an ultimate load (N_u) of 935 kN and hence a design value of $\varphi_s N_u$=841 kN. This compares with the column strength design value obtained using AS/NZS4600 of $\varphi_s N_c$=701 kN, where N_c=825.7 kN and φ_c=0.85. Thus, the axial design capacity obtained using GMNIA analysis is higher than that obtained using AS/NZS4600, which is partly because the system resistance factor (φ_s=0.9) is higher than the column resistance factor (φ_c=0.85), and partly because the nominal strength determined using GMNIA analysis is higher than the design strength obtained using the column strength curve in AS/NZS4600. The latter result may, in part, be a consequence of the omission of residual stresses in the GMNIA analysis model.

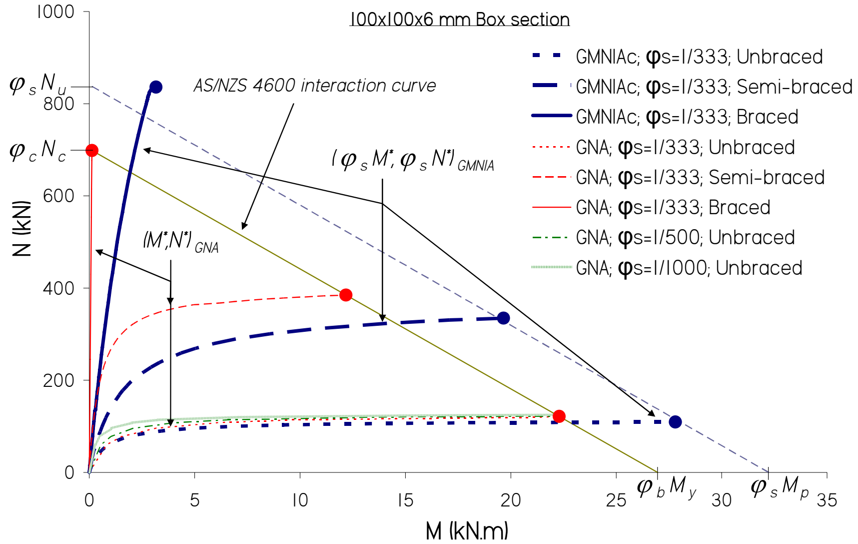
Table 6: Design capacities (Pu) for each bracing configuration

Bracing	Upright cross-section	Compact/ non-compact	Torsion of uprights in GMNIA	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/1000	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/500	$\frac{P_{u,LA}}{P_{u,GMNIA}}$ ϕ_s=1/333	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/1000	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/500	$\frac{P_{u,GNA}}{P_{u,GMNIA}}$ ϕ_s=1/333
unbraced	SHS	compact	no	1.08	1.09	1.11	1.07	1.08	1.10
	RF11015	compact	no	1.04	1.07	1.10	1.03	1.06	1.08
	RF11015	compact	yes	1.01	1.04	1.07	1.00	1.02	1.05
	RF11015	non-comp	yes	1.18	1.24	1.28	1.16	1.22	1.26
Average unbraced				**1.08**	**1.11**	**1.14**	**1.07**	**1.10**	**1.12**
braced	braced	compact	no	0.83	0.84	0.86	0.83	0.84	0.86
	braced	compact	no	0.87	0.88	0.89	0.87	0.88	0.89
	braced	compact	yes	0.79	0.80	0.81	0.79	0.80	0.81
	braced	non-comp	yes	0.83	0.84	0.85	0.83	0.84	0.85
Average braced				**0.83**	**0.84**	**0.85**	**0.83**	**0.84**	**0.85**
semi-braced	semi-brac	compact	no	1.18	1.18	1.19	1.15	1.16	1.17
	semi-brac	compact	no	1.18	1.19	1.20	1.11	1.12	1.13
	semi-brac	compact	yes	0.89	0.90	0.90	0.89	0.90	0.90
	semi-brac	non-comp	yes	1.00	1.01	1.01	1.00	1.01	1.01
Average semi-braced				**1.06**	**1.07**	**1.08**	**1.04**	**1.05**	**1.05**

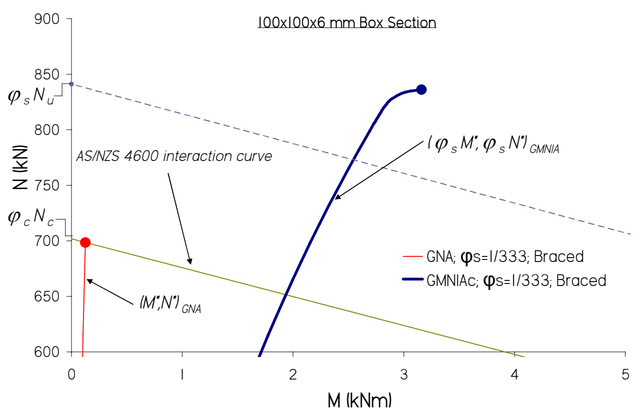
To investigate the effect of bending moments on the strength of rack frames, the (N^*,M^*)-values obtained from the GNA and GMNIAc analyses of the braced, unbraced and semi-braced frames with box section uprights are shown in Fig. 11 and compared with the linear interaction strength curve specified in AS/NZS4600. The following conclusions can be drawn from the figure:

- The bending moment in the critical upright (2nd upright from the left between the floor and the first beam level) of the braced frame is negligible in the GNA analysis (see Figs 11a and 11b) and smaller than in the GMNIAc analysis in which it is amplified by member imperfections. The axial capacity as obtained from GMNIAc analysis is insignificantly reduced by the presence of a bending moment in the braced frame, implying that the interaction curve determined by GMNIAc analysis is not linear in the high axial force region; a well-known result for compact I- and rectangular hollow sections, e.g. see [12].
- The (N^*,M^*)-curves are highly non-linear for the unbraced frames. As shown in Fig. 11c, the bending moment increases rapidly as the load (P) approaches the buckling load of the frame (P_c=20.9 kN, see Table 4) and, in effect, the design load (P_u) is governed by the elastic buckling load. Because this is

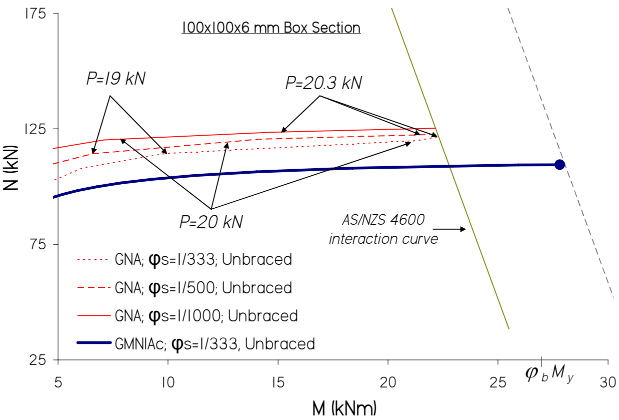
not factored down by a resistance factor, the design capacities based on LA and GNA analyses are higher than those based on GMNIAc analysis, which are always reduced by a system resistance factor (φ_s) irrespective of the mode of failure. This explains why the GMNIAc analysis-based design capacities shown in Tables 5 and 6 are conservative compared to LA and GNA analysis-based design capacities for unbraced frames.



(a) Results for braced, unbraced and semi-braced frames



(b) Results for braced frame at high axial loads



(c) Results for unbraced

Figure 11: (N,M)-paths for GNA and GMNIA analyses for braced rack frame.

It can be seen from the averages shown in Table 6 that for braced and semi-braced frames, the difference in the design capacities based on LA and GNA analyses is of the order of 1%-2% for out-of-plumb values varying from 1/1000 to 1/333. For unbraced frames, the design capacities based on LA and GNA analyses change by 6% and 5%, respectively, for out-of-plumb values varying from 1/1000 to 1/333; implying a modest dependency on the out-of-plumb.

CONCLUSIONS

This report presents a comparison of the design capacities of steel rack frames based on linear analysis (LA), geometric nonlinear analysis (GNA) and geometric and material nonlinear analysis (GMNIA). When based on LA and GNA analyses, the design is carried out to the Australian cold-formed steel structures AS/NZS4600. The study includes braced, unbraced and semi-braced frames. It is shown that,

- LA and GNA analyses produce nearly the same design capacities irrespective of the bracing configuration.
- On average, considering all bracing configurations, the design capacities based on LA and GNA analyses are within 1% of those determined using GMNIA analysis.

- GMNIA-based design is conservative for unbraced frames while optimistic for braced frames compared to design capacities based on LA and GNA analyses.
- The design capacity is insignificantly affected by out-of-plumb for braced and semi-braced frames, while moderately affected for unbraced frames, for which an increase in out-of-plumb from 1/1000 to 1/333 results in an average decrease in capacity of the order of 5%.
- Flexural-torsional buckling is shown to significantly reduce the design capacity in the case of rear-flange uprights subject to high axial forces.
- The rear-flange section is subject to distortional buckling. The GMNIA analysis-based design capacities are more significantly reduced than predicted by the Direct Strength Method incorporated in AS/NZS4600.

The study provides evidence to show that the structural design of steel rack frames may be based on advanced material and geometric nonlinear analyses. Such GMNIA analyses obviate the need for checking the section and/or member capacities to a structural standard. The study includes to compact and non-compact sections and members which fail by flexural as well as flexural-torsional buckling.

REFERENCES

[1] AS4100, *Steel Structures, AS4100*, Standards Australia: Sydney (1998)

[2] AISC, *Specification for Structural Steel Buildings, ANSI/AISC 360-05*, American Institute of Steel Construction: Chicago (2005)

[3] EN1993-1-1, *Eurocode 3: Design of steel structures Part 1-1: General rules and rules for buildings*, European Committee for Standardisation: Brussels (2005)

[4] Strand7, *Theoretical Manual, Version 2.4*, Sydney: G&D Computing (2009)

[5] Clarke, M.J., Bridge, R.Q., Hancock, G.J., and Trahair, N.S., Advanced analysis of steel building frames, *Journal of Constructional Steel Research* 23(1-3) (1992) 1-29.

[6] AS4084, *Steel Storage Racking*, Standards Australia: Sydney (1993)

[7] DB062/B0811, *Steel Storage Racking*, Standards Australia: Sydney (2008)

[8] Rasmussen, K.J.R. and Gilbert, B.P., *Analysis-based design provisions of steel storage racks*, Research Report 909, School of Civil Engineering, University of Sydney (2009)

[9] Hibbitt, Karlsson, and Sorensen, *Abaqus Standard User Manual, Version 6.3-1* (2002)

[10] Papangelis, J.P. and Hancock, G.J., Computer Analysis of Thin-Walled Structural Members, *Computers and Structures* 56(1) (1995.) 157-176

[11] AS/NZS4600, *Cold-formed Steel Structures, AS/NZS 4600*, Standards Australia: Sydney (2005)

[12] Trahair, N.S., Design strengths of steel beam-columns, *Canadian Journal of Civil Engineering,* 13(6) (1986) 639-646

APPENDIX 1

Rack buckling modes, buckling lengths and calculation of the ultimate design capacities of members

100x100x6 mm SHS

Unbraced rack – Compact cross-section and torsion of uprights ignored

Steel Storage Racks

Design Example: Unbraced rack
Compact SHS and CHS cross-section, (no locol or distortional buckling)
Down-aisle displacements only, (2D behaviour)
Flexure only, (no torsion)

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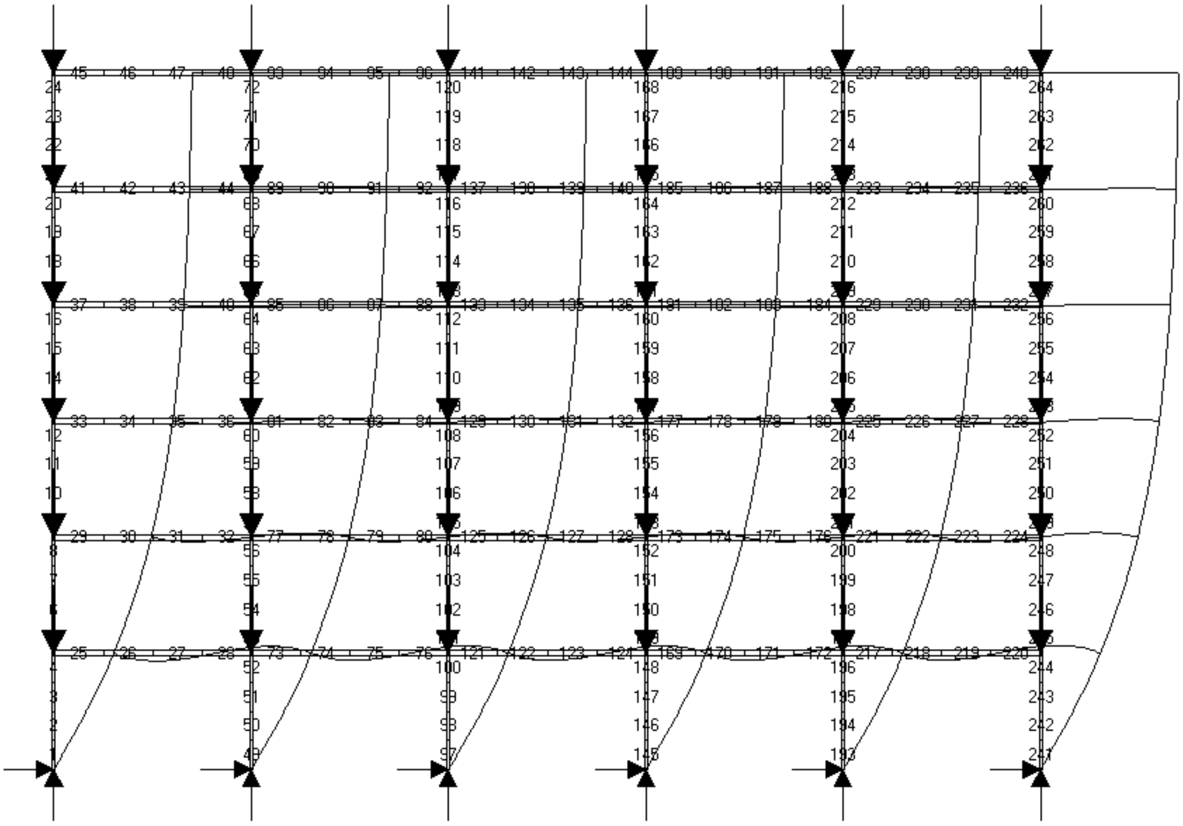


Fig. 1: Unbraced rack, box sections, element numbers and critical buckling mode (LBA)

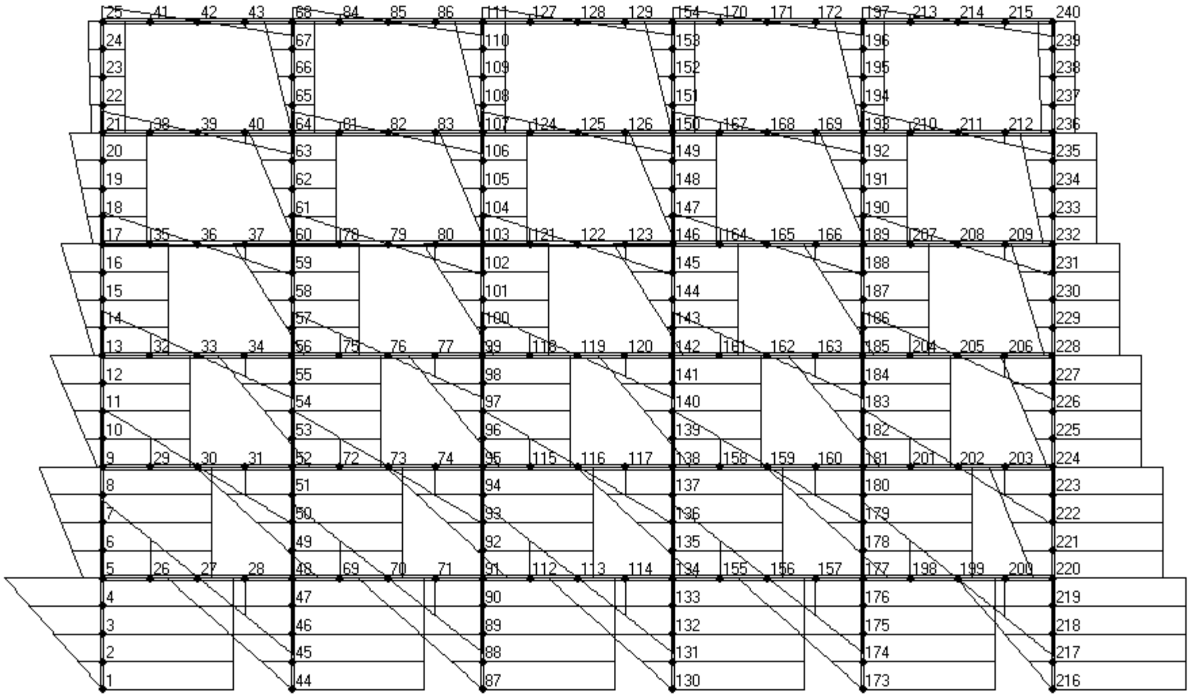


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

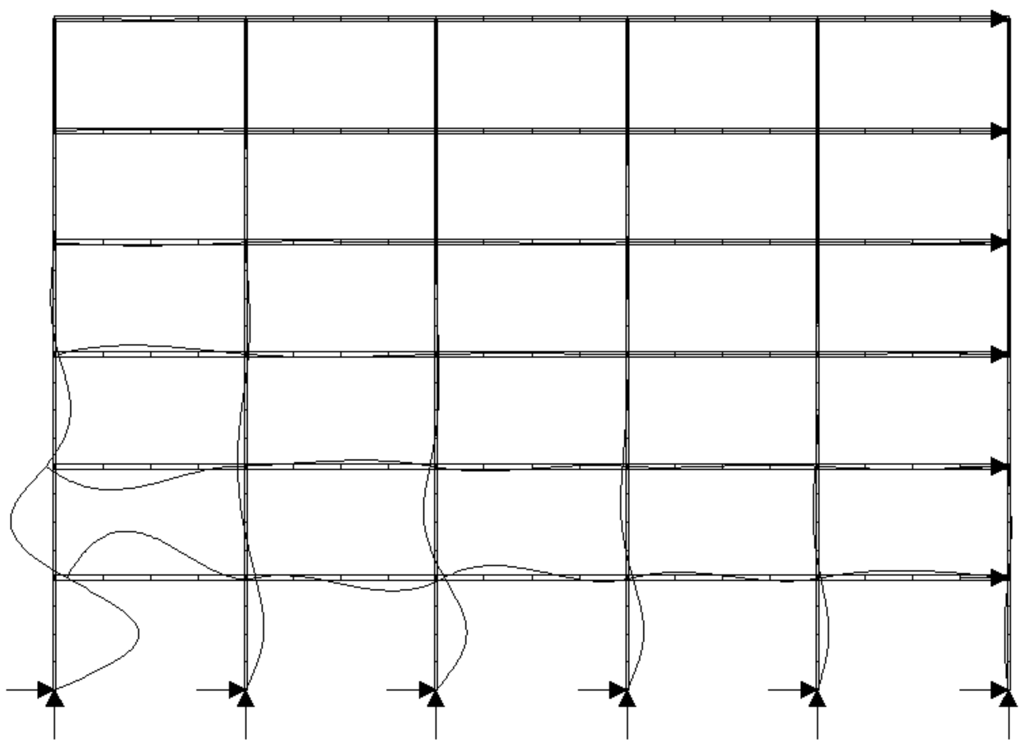


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The unbraced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 SHS100x100x6, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for an unbraced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$b_u := 100mm \quad t_u := 6mm \quad r_{ou} := 6 \cdot mm$

$A_u := 2256mm^2 \quad I_u := 3.336 \cdot 10^6 \cdot mm^4 \quad r_{iu} := r_{ou} - t_u$

$r_{iu} = 0\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}} \quad r_u = 38.454\,mm$

$Z_u := \frac{I_u}{\frac{b_u}{2}}$

Note: b_u, t_u, r_{ou} and r_{iu} are the width, thickness, outer corner radius and inner corner radius of the chord, respectively. A_u and I_u are the area and 2nd moment of area of the chord respectively.

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}} \quad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa \quad \nu := 0.3$

1 Design based on LA analysis

The maximum bending moment develops at node 48 in Element 52 at the first beam level, as shown in Fig. 2.The maximum axial force develops in Element 244 of the rightmost upright.

The axial force and bending moments in the critical uprights between the floor and 1st beam level (here termed Members 1 and 2), as determined from an LA analysis. are:

Member 1: N=-6.001P M_{11}= 0 M_{12}=-0.0391 P*m (Element 244 in LA, rightmost upright)
Member 2: N=-6.045P M_{21}= 0 M_{22}=-0.0225 P*m (Element 52 in LA, 2nd upright from left)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 20.94kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 342.8kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 20.94\text{kN}$

$c_{N1} := 6.001 \qquad N_{cr1} := c_{N1}\cdot P_{cr} \qquad N_{cr1} = 125.661\,\text{kN}$

$c_{N2} := 6.045 \qquad N_{cr2} := c_{N2}\cdot P_{cr} \qquad N_{cr2} = 126.582\,\text{kN}$

$P_{crb} := 342.8\cdot\text{kN} \qquad N_{crb} := 6\cdot P_{crb} \qquad N_{crb} = 2.057\times 10^{3}\,\text{kN}$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi\cdot\sqrt{\frac{E\cdot I_u}{N_{crb}}} \qquad L_e = 1.833\text{ m}$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 911.702\,\text{MPa}$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.703$$

$$f_n := \text{if}\left(\lambda_c < 1.5, 0.658^{\lambda_c^{2}}\cdot f_{yu}, \frac{0.977}{\lambda_c^{2}}\cdot f_{yu}\right) \qquad f_n = 366.009\,\text{MPa}$$

Determine the effective area:

$$b := b_u - 2r_{ou} \qquad b = 88\,\text{mm}$$

$$f_{cr} := \frac{4\cdot\pi^2\cdot E}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_u}{b}\right)^2 \qquad f_{cr} = 3.529\times 10^{3}\,\text{MPa}$$

$$\lambda := \sqrt{\frac{f_n}{f_{cr}}} \qquad \lambda = 0.322$$

$$b_{eu} := \text{if}\left[\lambda < 0.673, b, \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)\cdot b\right] \qquad b_{eu} = 88\,\text{mm}$$

$$A_{eu} := 4\cdot b_{eu}\cdot t_u + 4\cdot t_u^{\,2} \qquad A_{eu} = 2.256 \times 10^3\,\text{mm}^2$$

Column capacity:

$$N_c := A_{eu}\cdot f_n \qquad N_c = 825.717\,\text{kN}$$

Bending capacity of upright

The upright members will not fail by flexural-torsional buckling because of their high torsional rigidity. We therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$$M_{su} := f_{yu}\cdot Z_u \qquad M_{su} = 30.024\,\text{kN}\cdot\text{m}$$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_m M^*/(\phi_b\, M_b \alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad \Rightarrow \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}{*}P$, $c_{N1}=6.001$, $M_{11}{*}=0$ and $M_{12}{*}=c_{M1}{*}P{*}m$, $c_{M1}=-0.0391m$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{M1} := 0.0391\cdot\text{m}$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 20.94\,\text{kN}$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 20.2\,\text{kN}$$

$$\text{check} := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad \text{check} = 1$$

Member 2:

We have $N^*=c_{N2}{*}P$, c_{N2}=6.045, $M_{21}{*}$=0 and $M_{22}{*}=c_{M2}{*}P{*}m$, c_{M2}=-0.0225; and $\alpha_n=1-N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{M2} := 0.0225 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_c} + \frac{c_{M2} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2}\right) \qquad P_2 = 20.506\,\text{kN}$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of Members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 20.2\,\text{kN} \qquad P_2 = 20.506\,\text{kN}$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 20.2\,\text{kN} \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

In the GNA analysis, the maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity ($M_b=M_s$) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

$Data := $ GNA - unbraced - PRFSA.xls

$$Data = \begin{pmatrix} 16 & 2.498 & 98.42 & 3.152 & 96.05 \\ 18 & 4.832 & 112.4 & 6.135 & 108.1 \\ 20 & 15.91 & 133.7 & 21.31 & 119.9 \\ 20.3 & 21.53 & 140.6 & 29.77 & 121.6 \end{pmatrix}$$

$$P := \text{for } i \in 0..3 \quad \left| \begin{array}{l} ss_i \leftarrow Data_{i,0} \cdot kN \\ ss \end{array} \right.$$

Element 244 (right-hand upright):

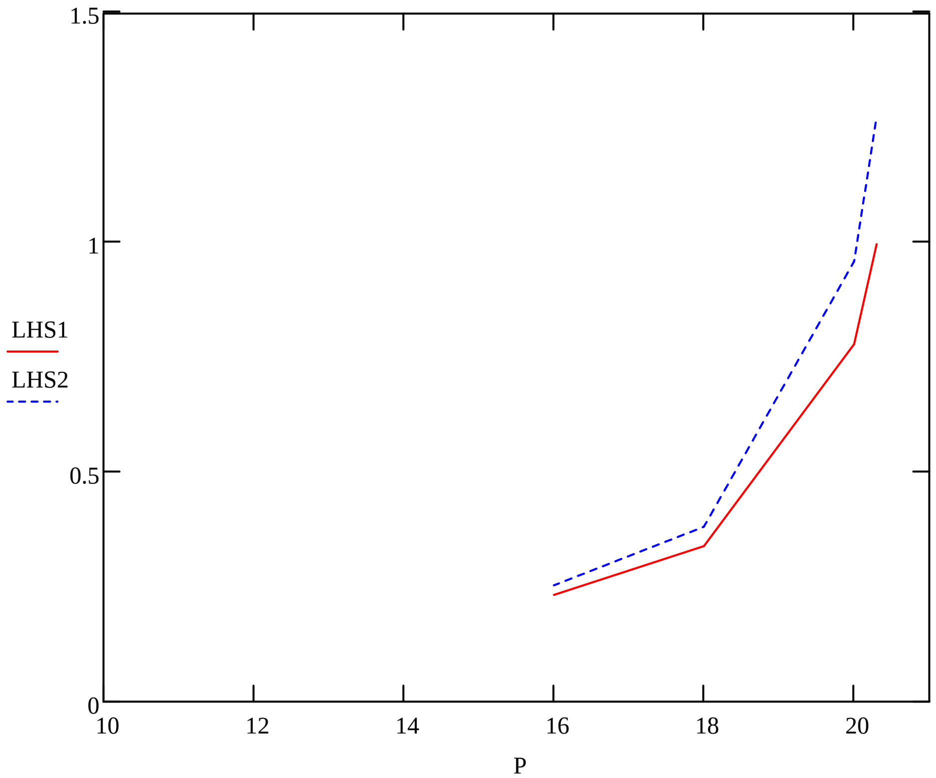
$$LHS1 := \text{for } i \in 0..3 \quad \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 16 \\ 18 \\ 20 \\ 20.3 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.233 \\ 0.339 \\ 0.779 \\ 0.997 \end{pmatrix}$$

Element 52 (2nd left-most upright):

$$LHS2 := \text{for } i \in 0..3 \quad \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 16 \\ 18 \\ 20 \\ 20.3 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.253 \\ 0.381 \\ 0.959 \\ 1.275 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 2$ $\quad x_1 := P_{n_u}$ $\quad x_2 := P_{n_u+1}$ $\quad y_1 := LHS2_{n_u}$ $\quad y_2 := LHS2_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 20\,kN \qquad y_1 = 0.959$$

$P_u = 20.039\,kN$ $\qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 20.3 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max}$ $\qquad P_{GMNIAc} = 18.27\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$P_{LA} = 20.2\,kN$ $\qquad P_{GNA} = 20.039\,kN$ $\qquad P_{GMNIAc} = 18.27\,kN$

The factored ultimate load (18.27kN) determined on the basis of a GMNIAc analysis is 10.6% and 9.7% lower than those (20.2kN and 20.039kN) obtained using LA and GNA analyses, respectively.

100x100x6 mm SHS

Semi-braced rack – Compact cross-section and torsion of uprights ignored

Steel Storage Racks

Design Example: Semi-braced rack
Compact SHS and CHS cross-section, (no locol or distortional buckling)
Down-aisle displacements only, (2D behaviour)
Flexure only, (no torsion)

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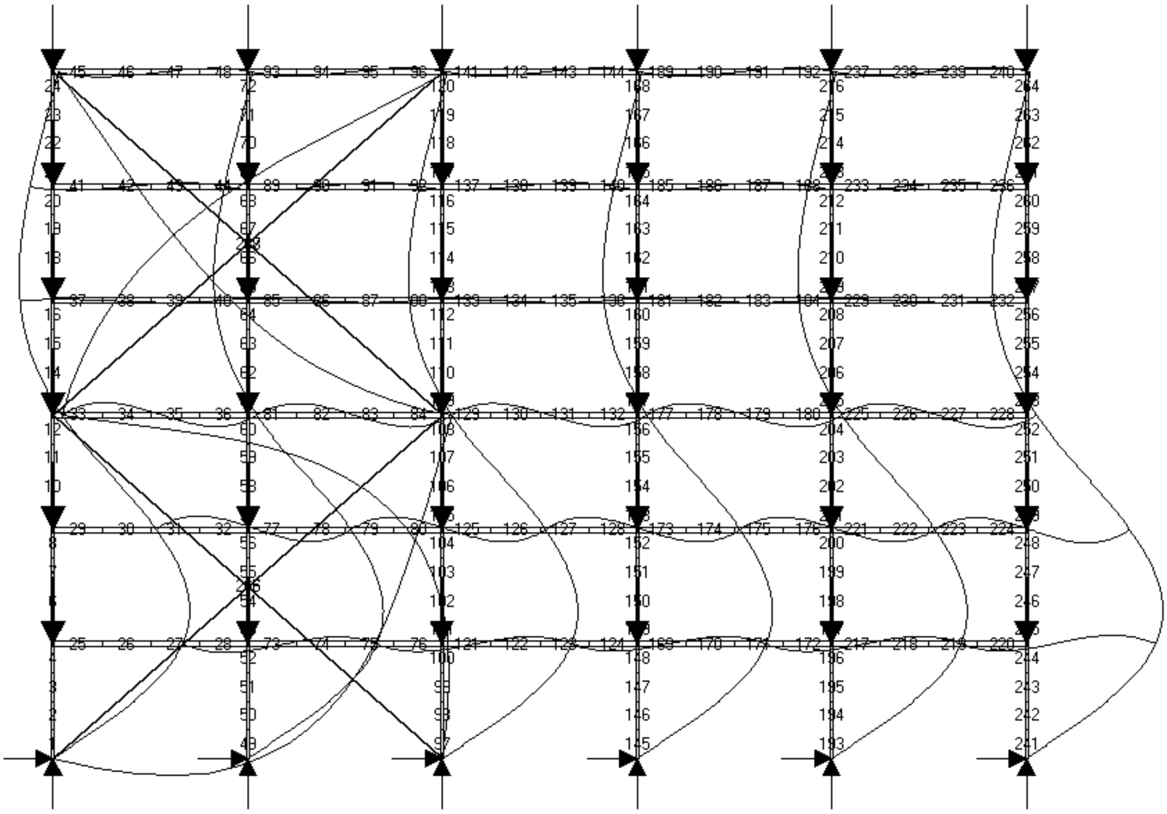


Fig. 1: Semi-braced rack, box sections, element numbers and critical buckling mode (LBA)

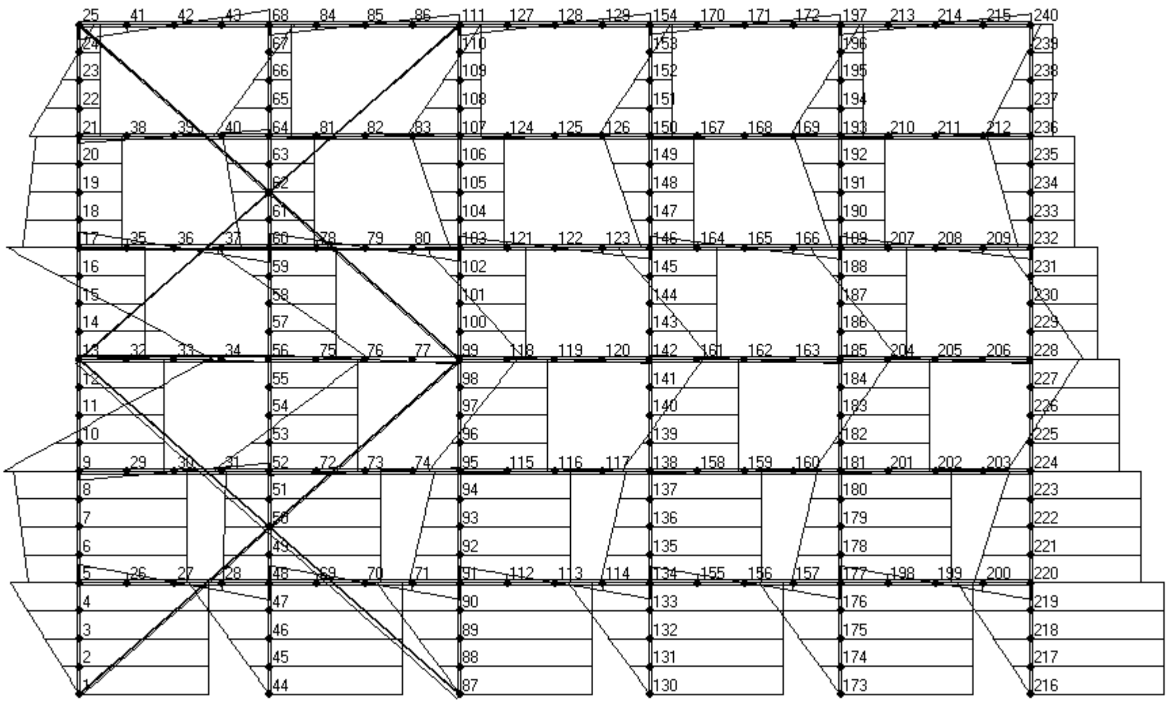


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

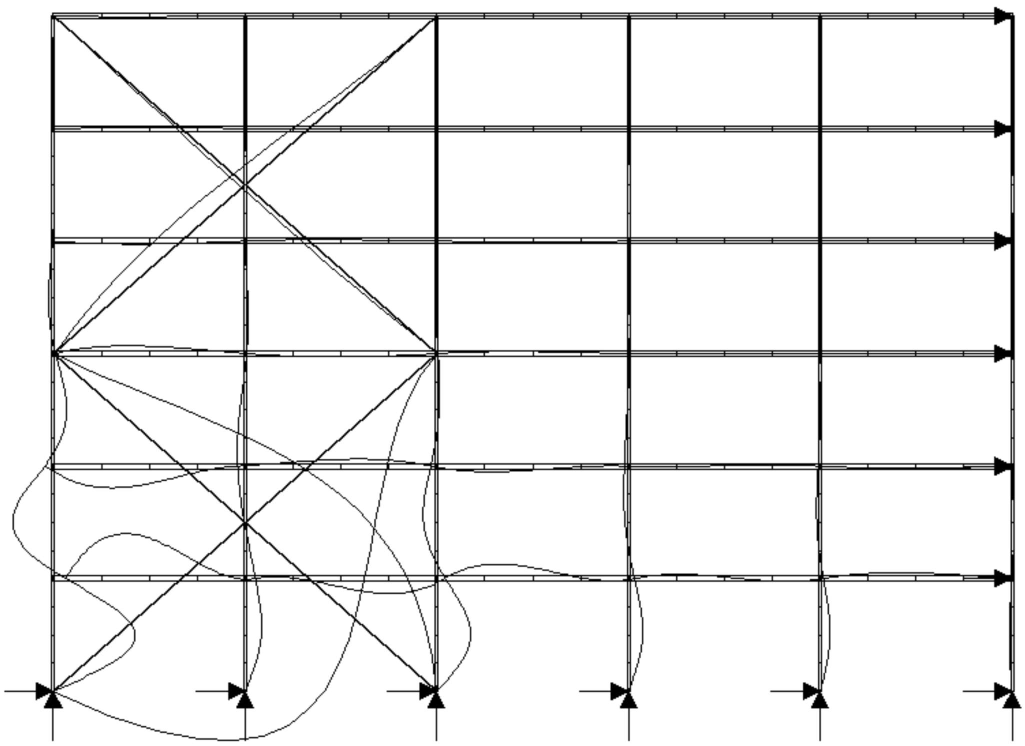


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The semi-braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. Bracing spans over three beam levels and so the frame is termed "semi-braced". The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 SHS100x100x6, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for an semi-braced steel storage rack.

Units:

$m := 1L$ $\quad sec := 1T$ $\quad kg := 1M$ $\quad mm := \frac{m}{1000}$ $\quad N := 1M \cdot 1\frac{L}{1T^2}$ $\quad MPa := \frac{N}{mm^2}$ $\quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$b_u := 100mm$ $\quad t_u := 6mm$ $\quad r_{ou} := 6 \cdot mm$

$A_u := 2256mm^2$ $\quad I_u := 3.336 \cdot 10^6 \cdot mm^4$ $\quad r_{iu} := r_{ou} - t_u$

$r_{iu} = 0\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}}$ $\quad r_u = 38.454\,mm$

$Z_u := \frac{I_u}{\frac{b_u}{2}}$

Note: b_u, t_u, r_{ou} and r_{iu} are the width, thickness, outer corner radius and inner corner radius of the chord, respectively. A_u and I_u are the area and 2nd moment of area of the chord respectively.

Beam geometry:

$b_b := 60mm$ $\quad t_b := 4mm$ $\quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2$ $\quad I_b := 4.707 \cdot 10^5 \cdot mm^4$ $\quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}}$ $\quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm$ $\quad t_s := 2mm$

$A_s := 175.9mm^2$ $\quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}}$ $\quad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ $\quad$ Beam $f_{yb} := 450MPa$ $\quad$ Brace $f_{ys} := 450MPa$

$E := 210000MPa$ $\quad \nu := 0.3$

1 Design based on LA analysis

The maximum axial force develops between the floor and the first beam level. The axial force is essentially the same in all uprights, although slightly lower in the left-most upright. Of the six uprights, the bending moment is fractionally higher at node 134 in Element 148. The maximum bending moment in the frame develops at node 13 in Element 13, as shown in Fig. 2.

The axial force and bending moments in the critical uprights between the floor and 1st beam level and in element 13 (here termed Members 1 and 2), as determined from an LA analysis. are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0052 P*m (Element 148 in LA, 4th upright from the left)
Member 2: N=-2.960P M_{21}= 0.0046 M_{22}=-0.0084 P*m (Element 13 in LA, leftmost upright)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 64.50kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 339.3kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 64.5kN$

$c_{N1} := 6.000$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 387\,kN$

$c_{N2} := 2.960$ $\quad N_{cr2} := c_{N2} \cdot P_{cr}$ $\quad N_{cr2} = 190.92\,kN$

$P_{crb} := 339.3 \cdot kN$ $\quad N_{crb} := 6 \cdot P_{crb}$ $\quad N_{crb} = 2.036 \times 10^3\,kN$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi \cdot \sqrt{\frac{E \cdot I_u}{N_{crb}}} \qquad L_e = 1.843\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 902.394\,MPa$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.706$$

$$f_n := \text{if}\left(\lambda_c < 1.5, 0.658^{\lambda_c^2} \cdot f_{yu}, \frac{0.977}{\lambda_c^2} \cdot f_{yu}\right) \qquad f_n = 365.23\,MPa$$

Determine the effective area:

$$b := b_u - 2r_{ou} \qquad b = 88\,mm$$

$$f_{cr} := \frac{4 \cdot \pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_u}{b}\right)^2 \qquad f_{cr} = 3.529 \times 10^3\,MPa$$

$$\lambda := \sqrt{\frac{f_n}{f_{cr}}} \qquad \lambda = 0.322$$

$$b_{eu} := \text{if}\left[\lambda < 0.673, b, \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)\cdot b\right] \qquad b_{eu} = 88\,\text{mm}$$

$$A_{eu} := 4\cdot b_{eu}\cdot t_u + 4\cdot t_u^2 \qquad A_{eu} = 2.256\times 10^3\,\text{mm}^2$$

Column capacity:

$$N_c := A_{eu}\cdot f_n \qquad N_c = 823.959\,\text{kN}$$

Bending capacity of upright

The upright members will not fail by flexural-torsional buckling because of their high torsional rigidity. We therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$$M_{su} := f_{yu}\cdot Z_u \qquad M_{su} = 30.024\,\text{kN}\cdot\text{m}$$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_m M^*/(\phi_b\, M_b\alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad \Rightarrow \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}{*}P$, $c_{N1}=6.000$, $M_{11}{*}=0$ and $M_{12}{*}=c_{M1}{*}P{*}m$, $c_{M1}=-0.0052m$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{M1} := 0.0052\cdot m$$

$$C_m := 1.0$$

$\phi_c := 0.85 \qquad \phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 64.5\,kN$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1} \right) \qquad P_1 = 62.812\,kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}} \right)} \qquad check = 1$$

Member 2:

We have $N^*=c_{N2}*P$, $c_{N2}=2.960$, $M_{21}*=c_{M21}*P*m$, $c_{M21}=0.0046$ and $M_{22}*=c_{M22}*P*m$, $c_{M22}=-0.0084$; and $\alpha_n=1-N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$c_{M21} := 0.0046 \cdot m \qquad c_{M22} := 0.0084 \cdot m$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_c} + \frac{c_{M22} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2} \right) \qquad P_2 = 62.786\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of Members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$P_1 = 62.812\,kN \qquad P_2 = 62.786\,kN$

$P_{min} := \min(P_1, P_2) \qquad P_{min} = 62.786\,kN \qquad P_{LA} := P_{min}$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level in the GNA analysis.The maximum axial force and bending moment are found in the 4th upright from the left (Element 196). The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity ($M_b=M_s$) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data := GNA - semibraced - PRFSA.xls

Data =

59	4.957	363.92
59.5	6.191	367.766
60	6.951	371.279
60.5	7.925	374.916
61	9.231	378.744
61.5	11.115	382.909
62	14.244	387.802
62.5	23.665	396.432

$P := \text{for } i \in 0..7$

$\quad ss_i \leftarrow Data_{i,0} \cdot kN$

$\quad ss$

Element 196 (4th upright from the left):

$LHS1 := \text{for } i \in 0..7$

$\quad N \leftarrow Data_{i,2} \cdot kN$

$\quad M \leftarrow Data_{i,1} \cdot kN \cdot m$

$$ss_i \leftarrow \frac{N}{\phi_c \cdot N_c} + \frac{M}{\phi_b \cdot M_{su}}$$

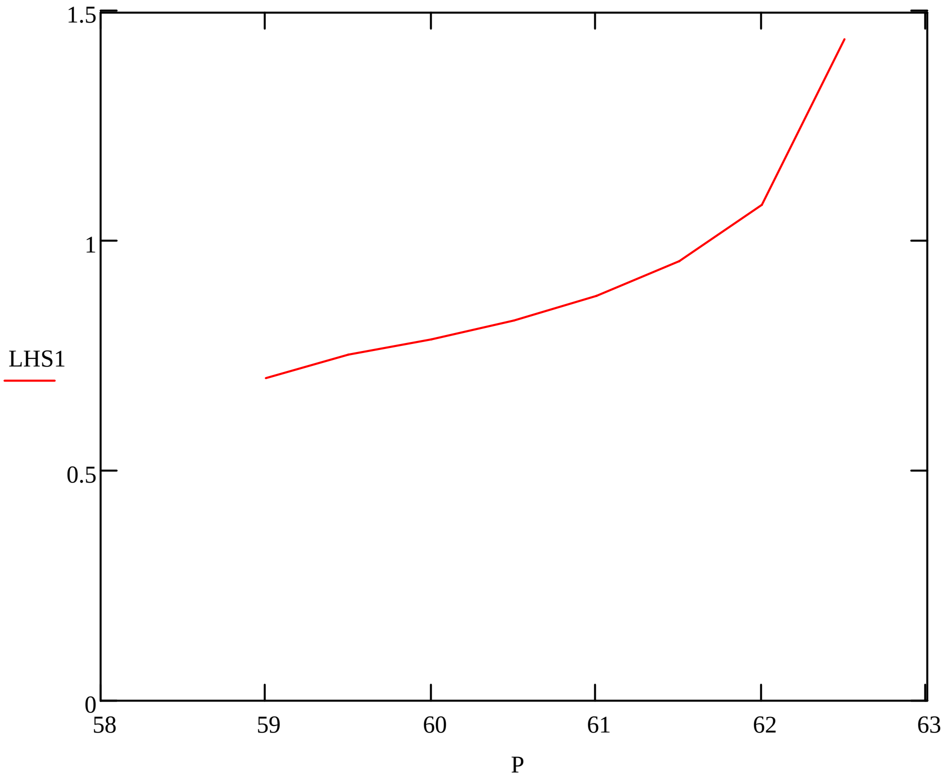
$\quad ss$

P = (kN)

59
59.5
60
60.5
61
61.5
62
62.5

LHS1 =

0.703
0.754
0.787
0.829
0.882
0.958
1.081
1.442



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 5$ $x_1 := P_{n_u}$ $x_2 := P_{n_u+1}$ $y_1 := LHS1_{n_u}$ $y_2 := LHS1_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1$$ $x_1 = 61.5\,kN$ $y_1 = 0.958$

$P_u = 61.671\,kN$ $P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 58.7 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max}$ $P_{GMNIAc} = 52.83\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$P_{LA} = 62.786\,kN$ $P_{GNA} = 61.671\,kN$ $P_{GMNIAc} = 52.83\,kN$

The factored ultimate load (52.83kN) determined on the basis of a GMNIAc analysis is 18.8% and 16.7% lower than those (62.786kN and 61.671kN) obtained using LA and GNA analyses, respectively.

100x100x6 mm SHS

Fully braced rack – Compact cross-section and torsion of uprights ignored

Design Example: Fully braced rack
Compact SHS and CHS cross-section, (no local or distortional buckling)
Down-aisle displacements only, (2D behaviour)
Flexure only, (no torsion)

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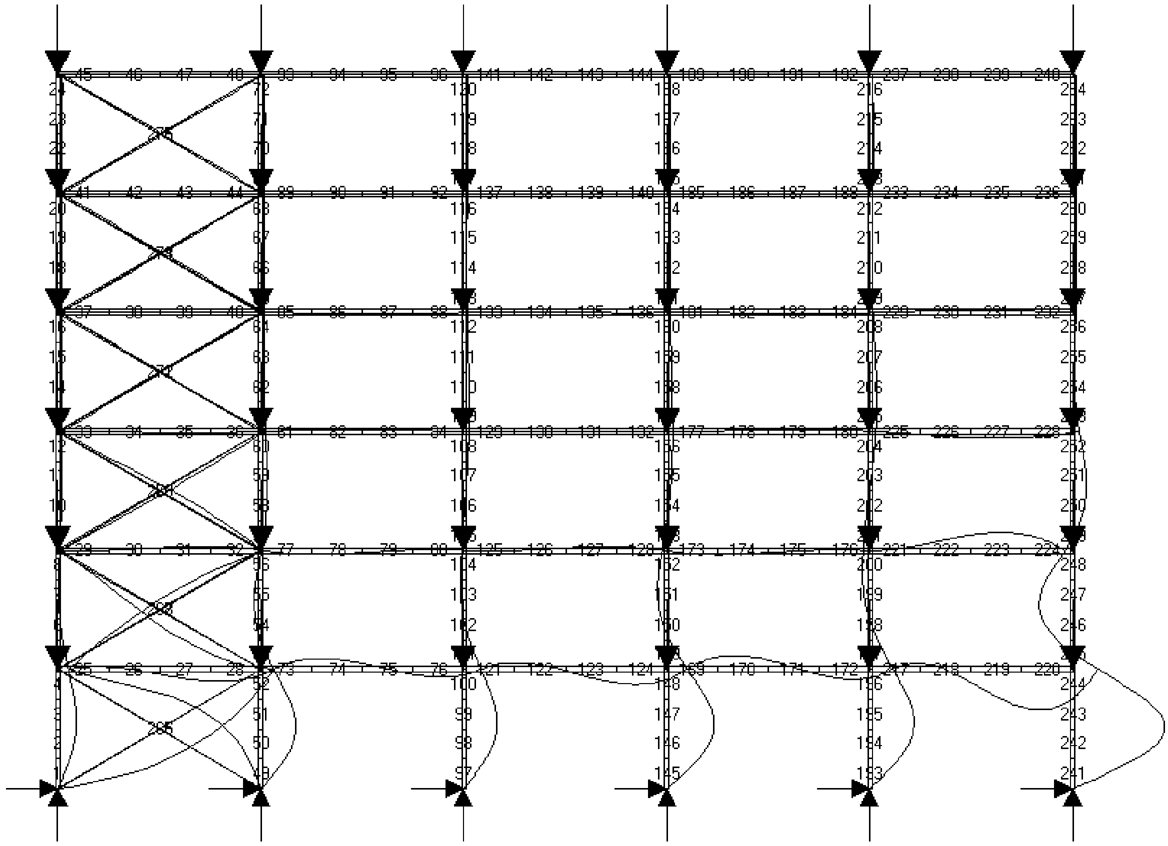


Fig. 1: Fully braced rack, box sections, element numbers and critical buckling mode (LBA)

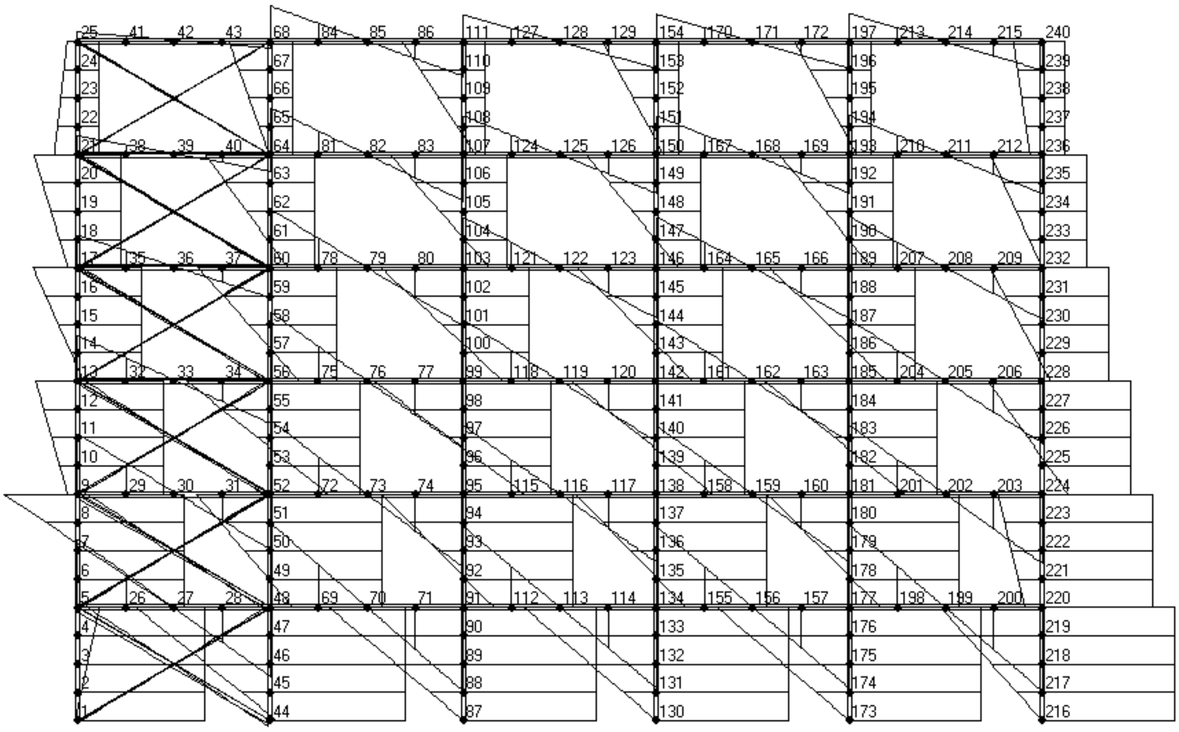


Fig. 2: Node numbers, and axial and bending moment diagrams (LA)

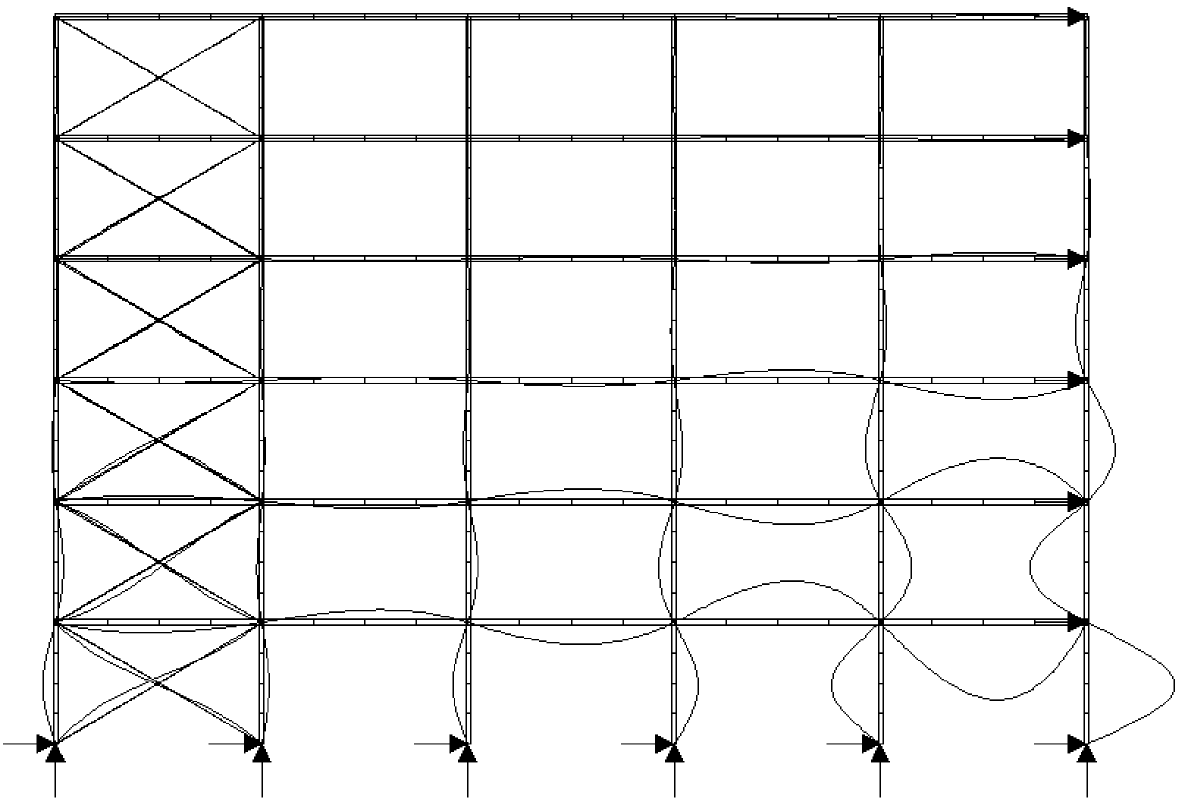


Fig. 3: Buckling mode when all beam levels are restrained (LBA)

Required: The fully braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 SHS100x100x6, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for a fully braced steel storage rack.

Units:

$m := 1L$ $\quad sec := 1T$ $\quad kg := 1M$ $\quad mm := \frac{m}{1000}$ $\quad N := 1M \cdot 1\frac{L}{1T^2}$ $\quad MPa := \frac{N}{mm^2}$ $\quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$b_u := 100mm$ $\quad t_u := 6mm$ $\quad r_{ou} := 6 \cdot mm$

$A_u := 2256mm^2$ $\quad I_u := 3.336 \cdot 10^6 \cdot mm^4$ $\quad r_{iu} := r_{ou} - t_u$

$r_{iu} = 0\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}}$ $\quad r_u = 38.454\,mm$

$Z_u := \frac{I_u}{\frac{b_u}{2}}$

Note: b_u, t_u, r_{ou} and r_{iu} are the width, thickness, outer corner radius and inner corner radius of the chord, respectively. A_u and I_u are the area and 2nd moment of area of the chord respectively.

Beam geometry:

$b_b := 60mm$ $\quad t_b := 4mm$ $\quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2$ $\quad I_b := 4.707 \cdot 10^5 \cdot mm^4$ $\quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}}$ $\quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm$ $\quad t_s := 2mm$

$A_s := 175.9mm^2$ $\quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}}$ $\quad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ $\quad$ Beam $f_{yb} := 450MPa$ $\quad$ Brace $f_{ys} := 450MPa$

$E := 210000MPa$ $\quad \nu := 0.3$

1 Design based on LA analysis

The maximum axial force and maximum bending moment develop at node 48 in Element 52 at the first beam level, as shown in Fig. 2.

The axial force and bending moment in the critical upright between the floor and 1st beam level (here termed Member 1), as determined from an LA analysis, are:

Member 1: N=-6.128P M_{11}= 0 M_{12}=-0.0011 P*m (Element 52 in LA)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 337.1kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 342.8kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 337.1\text{kN}$

$c_{N1} := 6.128$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 2.066 \times 10^{3}\,\text{kN}$

$P_{crb} := 342.8 \cdot \text{kN}$ $\quad N_{crb} := 6 \cdot P_{crb}$ $\quad N_{crb} = 2.057 \times 10^{3}\,\text{kN}$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi \cdot \sqrt{\frac{E \cdot I_u}{N_{crb}}} \qquad L_e = 1.833\ \text{m}$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 911.702\,\text{MPa}$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.703$$

$$f_n := \text{if}\left(\lambda_c < 1.5, 0.658^{\lambda_c^{2}} \cdot f_{yu}, \frac{0.977}{\lambda_c^{2}} \cdot f_{yu}\right) \qquad f_n = 366.009\,\text{MPa}$$

Determine the effective area:

$$b := b_u - 2r_{ou} \qquad b = 88\,\text{mm}$$

$$f_{cr} := \frac{4 \cdot \pi^{2} \cdot E}{12 \cdot \left(1 - \nu^{2}\right)} \cdot \left(\frac{t_u}{b}\right)^{2} \qquad f_{cr} = 3.529 \times 10^{3}\,\text{MPa}$$

$$\lambda := \sqrt{\frac{f_n}{f_{cr}}} \qquad \lambda = 0.322$$

$$b_{eu} := \text{if}\left[\lambda < 0.673, b, \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)\cdot b\right] \qquad b_{eu} = 88\,\text{mm}$$

$$A_{eu} := 4\cdot b_{eu}\cdot t_u + 4\cdot t_u^2 \qquad A_{eu} = 2.256 \times 10^3\,\text{mm}^2$$

Column capacity:

$$N_c := A_{eu}\cdot f_n \qquad N_c = 825.717\,\text{kN}$$

Bending capacity of upright

The upright members will not fail by flexural-torsional buckling because of their high torsional rigidity. We therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$$M_{su} := f_{yu}\cdot Z_u \qquad M_{su} = 30.024\,\text{kN}\cdot\text{m}$$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_m M^*/(\phi_b\, M_b \alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_N{*}P$, $c_N=6.128$, $M_{11}{*}=0$ and $M_{12}{*}=c_M{*}P{*}m$, $c_M=-0.0011$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{M1} := 0.0011\cdot m$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 337.1\ kN$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\ 2} - 4 \cdot AA_1} \right) \qquad P_1 = 113.733\ kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}} \right)} \qquad check = 1$$

Design capacity of storage rack based on LA analysis:

The maximum factored design load (P) is:

$$P_{LA} := P_1 \qquad P_{LA} = 113.733\ kN$$

2 Design based on GNA analysis

The maximum design actions develop near the base of the right-most upright. In the GNA analysis, the axial force (N) and bending moment (M) are nonlinear functions of the applied force (P), as shown in Figs 5 and 6 respectively.

The axial member capacity (N_c) and bending capacity (M_b=M_s) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\ N_c) + M^*/(\phi_b\ M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data := GNA - braced - PRFSA.xls

$$Data = \begin{pmatrix} 100 & 0.105 & 615.2 & 0.113 & 600 \\ 110 & 0.12 & 676.9 & 0.129 & 660 \\ 120 & 0.134 & 738 & 0.147 & 720 \end{pmatrix}$$

$$P := \text{for } i \in 0..2 \quad \left| \begin{array}{l} ss_i \leftarrow Data_{i,0} \cdot kN \\ ss \end{array} \right.$$

Element 52 (bottom of 2nd left-most upright):

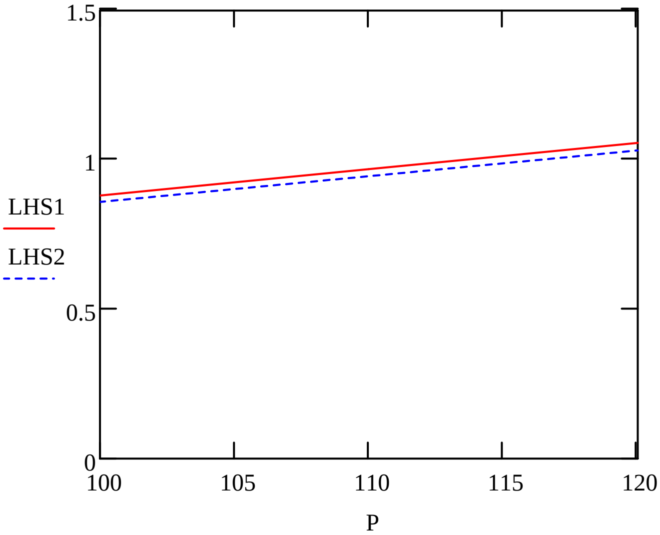
$$LHS1 := \text{for } i \in 0..2 \quad \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 100 \\ 110 \\ 120 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.88 \\ 0.969 \\ 1.056 \end{pmatrix}$$

Element 196 (bottom of 2nd right-most upright):

$$LHS2 := \text{for } i \in 0..2 \quad \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 100 \\ 110 \\ 120 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.859 \\ 0.945 \\ 1.031 \end{pmatrix}$$



The LHS of the interaction equation varies essentially linearly with the applied load (P) in the load range shown. Determine the value of P producing a LHS of unity by interpolation:

$n_u := 1 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS1_{n_u} \qquad y_2 := LHS1_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 110\,kN \qquad y_1 = 0.969$$

$P_u = 113.554\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$$P_{max} := 146.2 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 131.58\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$$P_{LA} = 113.733\,kN \qquad P_{GNA} = 113.554\,kN \qquad P_{GMNIAc} = 131.58\,kN$$

The factored ultimate load (131.58kN) determined on the basis of a GMNIAc analysis is 13.6% and 13.7% higher than those (113.733kN and 113.554kN) obtained using LA and GNA analyses, respectively.

RF11015

Unbraced rack – Compact cross-section and torsion of uprights ignored

Steel Storage Racks

Design Example: Unbraced rack
RF10015 section for uprights and SHS for pallet beams; all members analysed and designed assuming local and distortional buckling does not occur.
Down-aisle displacements only, (2D behaviour).
Flexure only, (while torsion of the uprights will occur in the ultimate limit state, torsion is ignored in the analysis and design calculations).

Kim Rasmussen & Benoit Gilbert

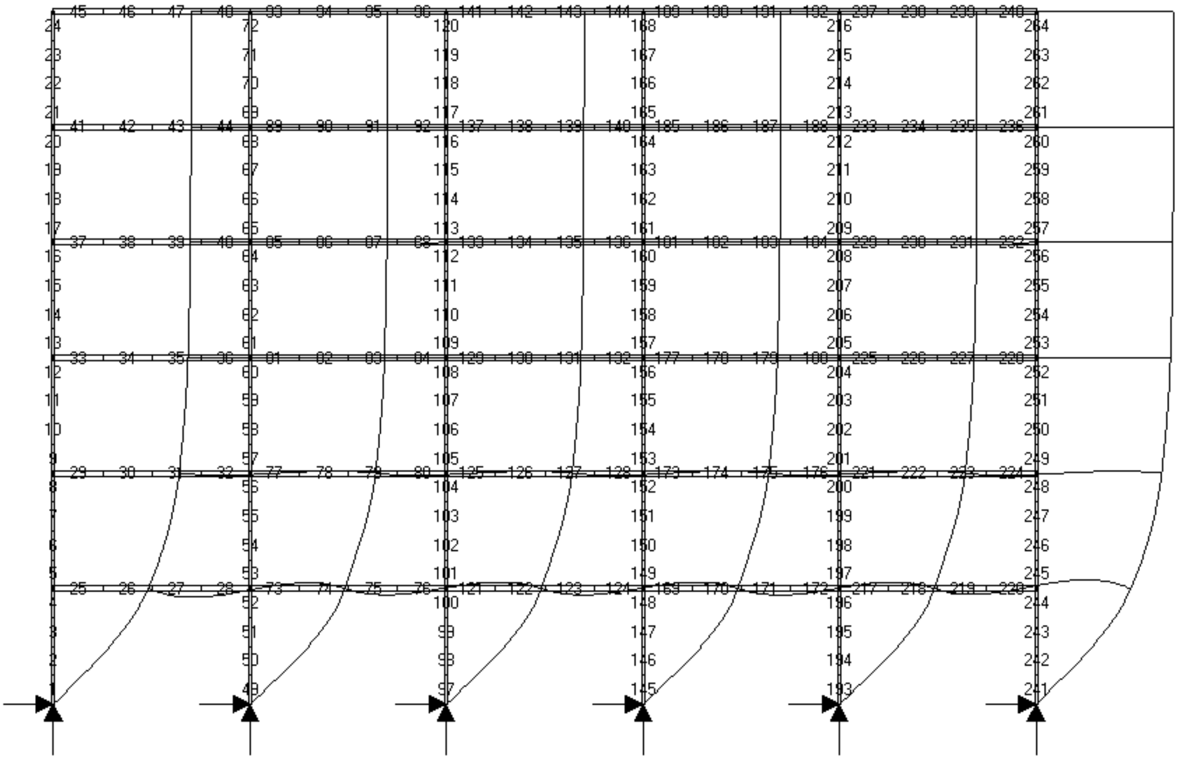


Fig. 1: Unbraced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

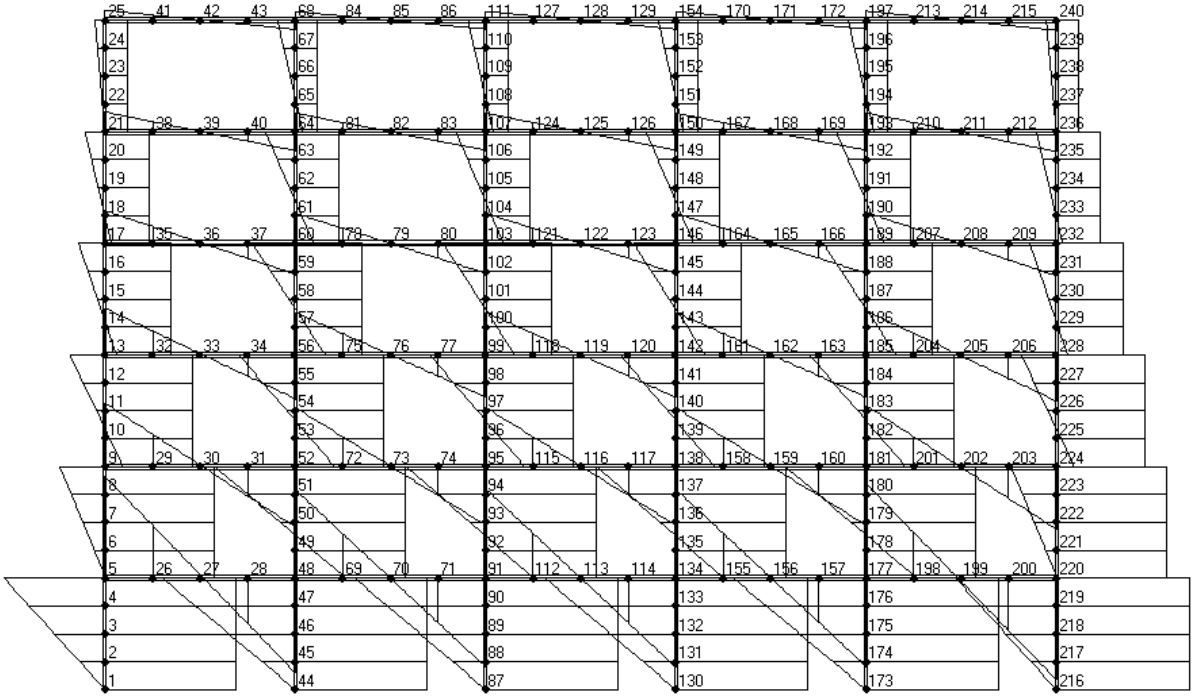


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

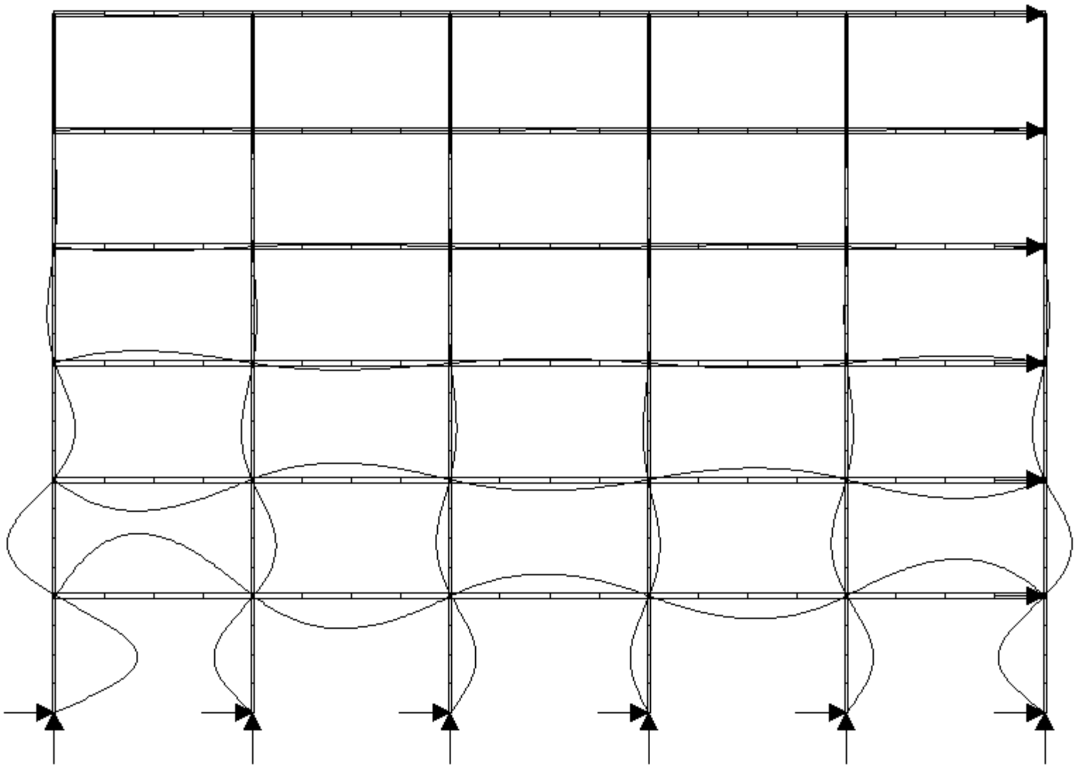


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The unbraced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF100115, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for an unbraced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$A_u := 508.5mm^2 \quad I_u := 8.484 \cdot 10^5 \cdot mm^4 \quad x_{max} := \frac{110}{2} \cdot mm \quad x_{max} = 55\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}} \quad r_u = 40.847\,mm$

$Z_u := \frac{I_u}{x_{max}}$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}} \quad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa \quad \nu := 0.3$

1 Design based on LA analysis

The maximum bending moment develops at node 48 in Element 52 at the first beam level, as shown in Fig. 2.The maximum axial force develops in Element 244 of the rightmost upright.

The axial force and bending moments in the critical uprights between the floor and 1st beam level (here termed Members 1 and 2), as determined from an LA analysis, are:

Member 1: N=-6.046P M_{11}= 0 M_{12}=-0.0299 P*m (Element 244 in LA, rightmost upright)
Member 2: N=-6.003P M_{21}= 0 M_{22}=-0.0394 P*m (Element 52 in LA, 2nd upright from left)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 11.05kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.43kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 11.05\text{kN}$

$c_{N1} := 6.046$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 66.808\,\text{kN}$

$c_{N2} := 6.003$ $\quad N_{cr2} := c_{N2} \cdot P_{cr}$ $\quad N_{cr2} = 66.333\,\text{kN}$

$P_{crb} := 95.43 \cdot \text{kN}$ $\quad N_{crb} := 6 \cdot P_{crb}$ $\quad N_{crb} = 572.58\,\text{kN}$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi \cdot \sqrt{\frac{E \cdot I_u}{N_{crb}}} \qquad L_e = 1.752\ \text{m}$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 1.126 \times 10^3\,\text{MPa}$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.632$$

$$f_n := \text{if}\left(\lambda_c < 1.5, 0.658^{\lambda_c^{\,2}} \cdot f_{yu}, \frac{0.977}{\lambda_c^{\,2}} \cdot f_{yu}\right) \qquad f_n = 380.687\,\text{MPa}$$

Determine the effective area:

Since the section is assumed not to undergo local or distortional buckling, the effective area is taken as the gross area.

$$A_{eu} := A_u \qquad A_{eu} = 508.5\,\text{mm}^2$$

Column capacity:

$$N_c := A_{eu} \cdot f_n \qquad N_c = 193.58\,\text{kN}$$

Bending capacity of upright

The upright members are assumed not to develop torsion. Accordingly, we therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$M_{su} := f_{yu} \cdot Z_u$ $\qquad M_{su} = 6.941\ kN \cdot m$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\ N_c) + C_m M^*/(\phi_b\ M_b \alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.046$, $M_{11}*=0$ and $M_{12}*=c_{M1}*P*m$, $c_{M1}=-0.0299m$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies C_m=0.85.

$c_{M1} := 0.0299 \cdot m$

$C_m := 1.0$

$\phi_c := 0.85$ $\qquad \phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 11.05\ kN$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 10.189\ kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have N*=c_{N2}*P, c_{N2}=6.003, M_{21}*=0 and M_{22}*=c_{M2}*P*m, c_{M2}=-0.0394; and α_n=1-N*/N_e. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{M2} := 0.0394 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_c \cdot P_{cr}} \qquad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_c} + \frac{c_{M2} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2} \right) \qquad P_2 = 9.96\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of Members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 10.189\,kN \qquad P_2 = 9.96\,kN$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 9.96\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

In the GNA analysis, the maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_b=M_s) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data :=
GNA - unbraced - PRFSA.xls

$$Data = \begin{pmatrix} 9 & 1.661 & 55.31 & 2.086 & 54.1 \\ 9.5 & 2.325 & 58.72 & 2.913 & 57.14 \\ 10 & 3.62 & 62.54 & 4.535 & 60.2 \\ 10.5 & 7.137 & 67.66 & 8.985 & 63.36 \end{pmatrix}$$

$$P := \text{for } i \in 0..3 \;\; \left| \begin{array}{l} ss_i \leftarrow Data_{i,0} \cdot kN \\ ss \end{array} \right.$$

Element 244 (right-hand upright):

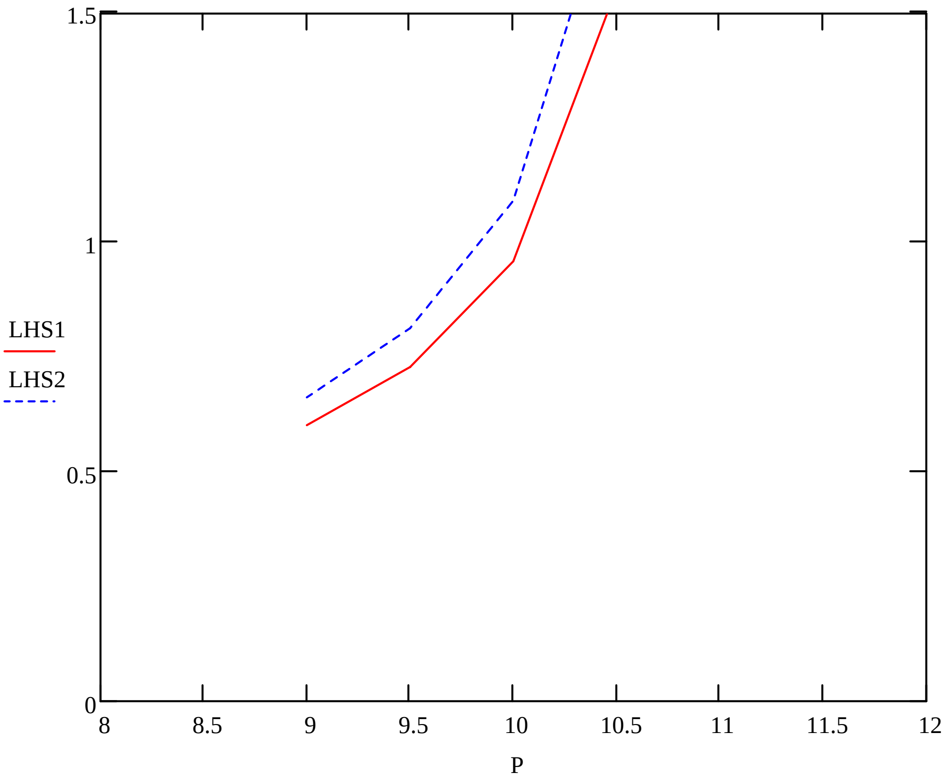
$$LHS1 := \text{for } i \in 0..3 \;\; \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.602 \\ 0.729 \\ 0.96 \\ 1.554 \end{pmatrix}$$

Element 52 (2nd left-most upright):

$$LHS2 := \text{for } i \in 0..3 \;\; \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.663 \\ 0.814 \\ 1.092 \\ 1.823 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 1$ $\quad x_1 := P_{n_u}$ $\quad x_2 := P_{n_u+1}$ $\quad y_1 := LHS2_{n_u}$ $\quad y_2 := LHS2_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 9.5\,kN \qquad y_1 = 0.814$$

$P_u = 9.835\,kN$ $\qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 10.1 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max}$ $\qquad P_{GMNIAc} = 9.09\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$P_{LA} = 9.96\,kN$ $\qquad P_{GNA} = 9.835\,kN$ $\qquad P_{GMNIAc} = 9.09\,kN$

The factored ultimate load (9.09kN) determined on the basis of a GMNIAc analysis is 9.6% and 8.2% lower than those (9.96kN and 9.835kN) obtained using LA and GNA analyses, respectively.

RF11015

Semi-braced rack – Compact cross-section and torsion of uprights ignored

Steel Storage Racks

Design Example: Semi-braced rack
RF10015 section for uprights and SHS for pallet beams; all members analysed and designed assuming local and distortional buckling does not occur.
Down-aisle displacements only, (2D behaviour).
Flexure only, (while torsion of the uprights will occur in the ultimate limit state, torsion is ignored in the analysis and design calculations).

Kim Rasmussen & Benoit Gilbert

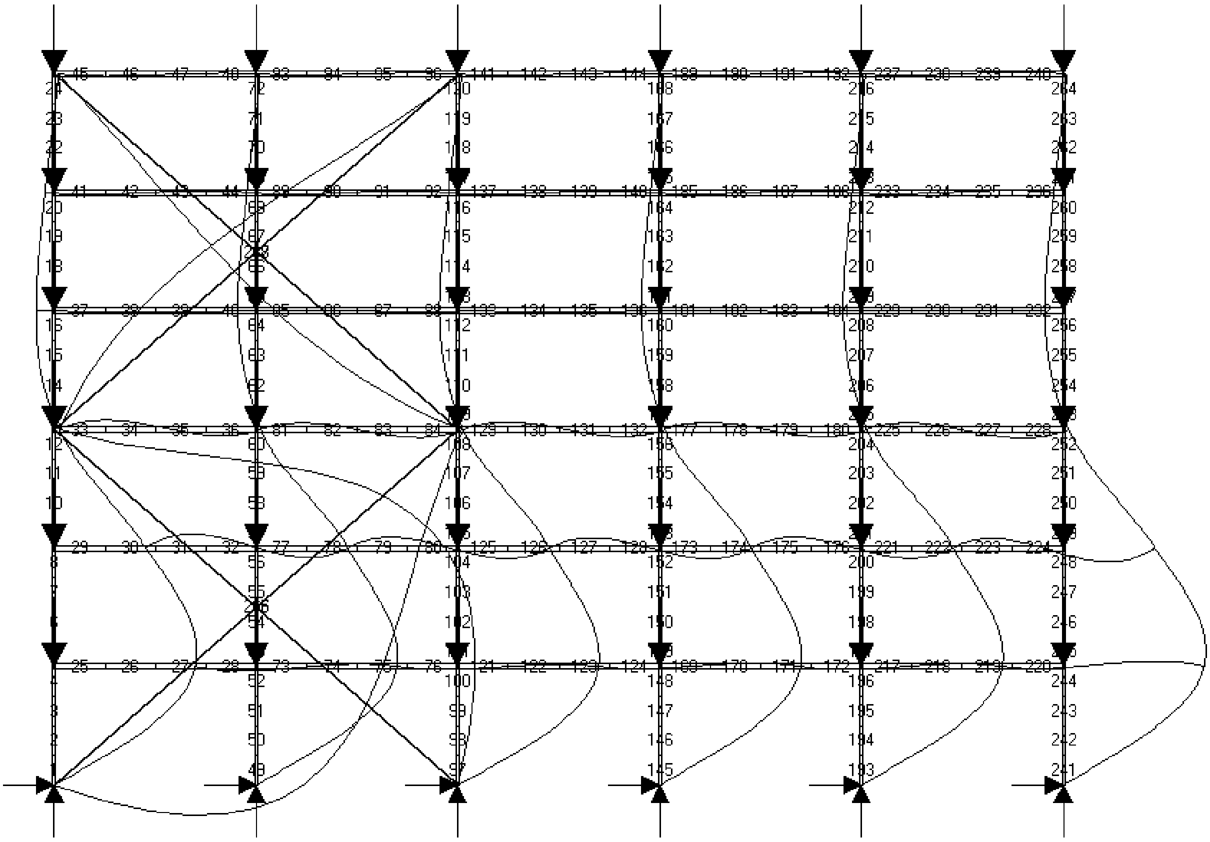


Fig. 1: Semi-braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

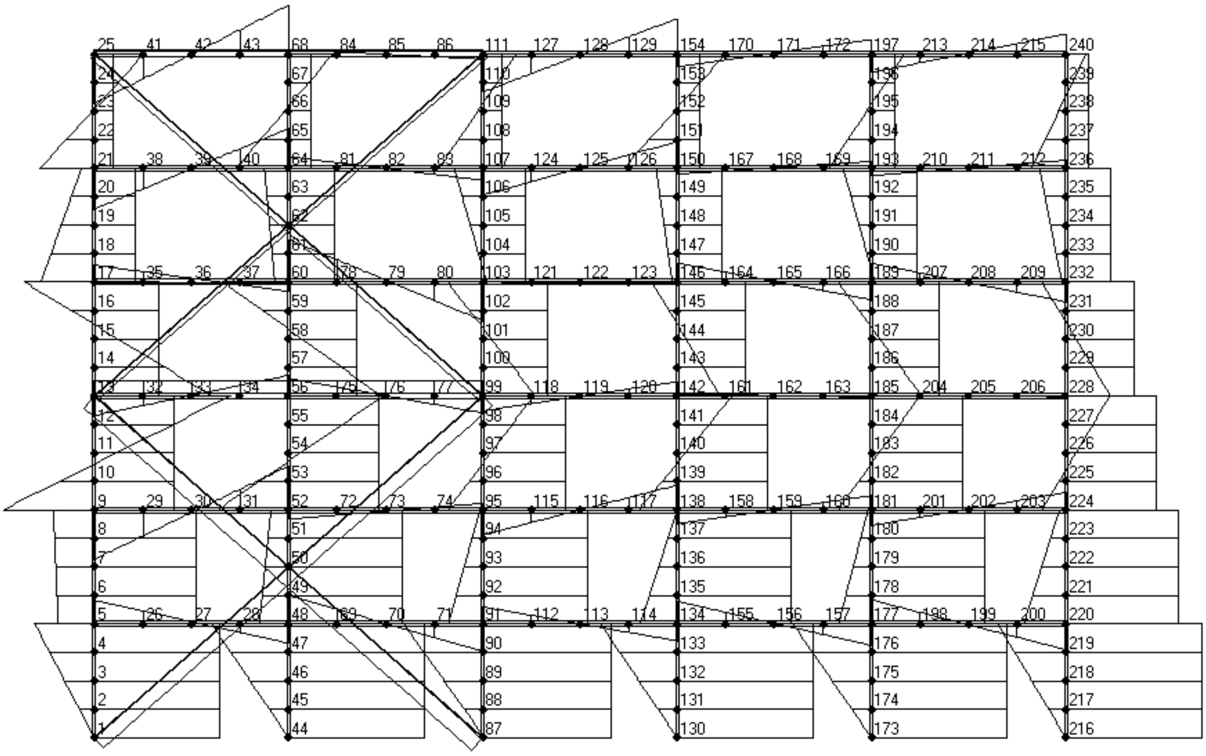


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

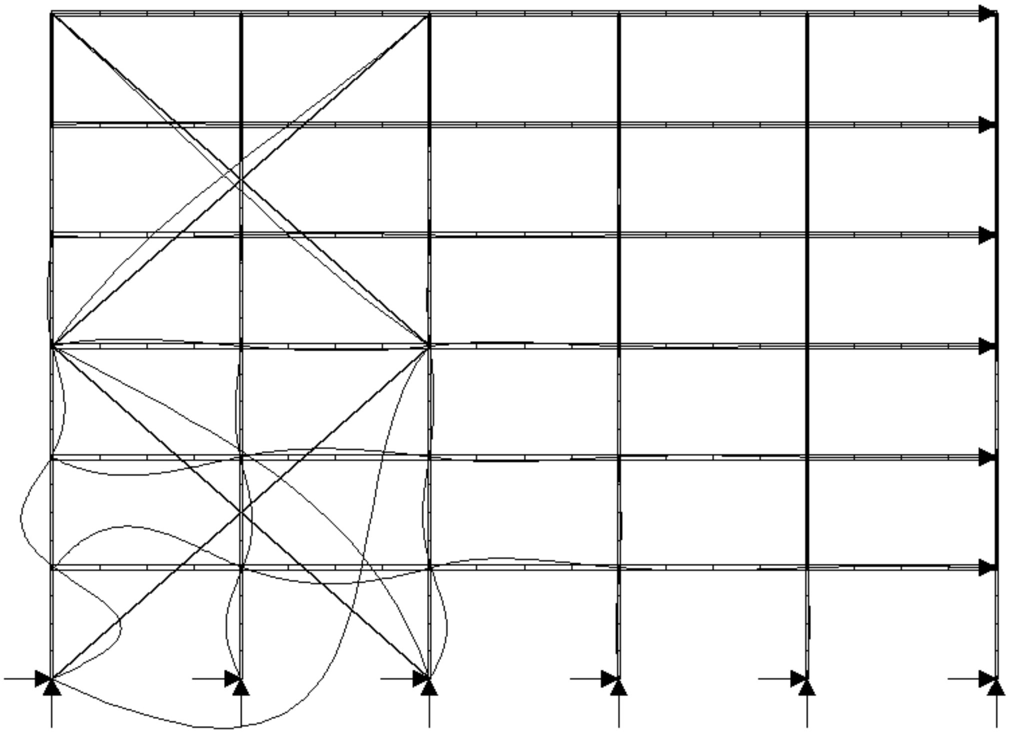


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

<u>Required:</u> The semi-braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. Bracing spans over three beam levels and so the frame is termed "semi-braced". The rack is assumed to be pin-ended at the

base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF100115, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for an semi-braced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$A_u := 508.5mm^2 \qquad I_u := 8.484 \cdot 10^5 \cdot mm^4 \qquad x_{max} := \frac{110}{2} \cdot mm \qquad x_{max} = 55\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}} \qquad r_u = 40.847\,mm$

$Z_u := \frac{I_u}{x_{max}}$

Beam geometry:

$b_b := 60mm \qquad t_b := 4mm$

$A_b := 896mm^2 \qquad I_b := 4.707 \cdot 10^5 \cdot mm^4$

$r_b := \sqrt{\frac{I_b}{A_b}} \qquad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \qquad t_s := 2mm$

$A_s := 175.9mm^2 \qquad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}} \qquad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa \qquad \nu := 0.3$

1 Design based on LA analysis

The maximum axial force develops between the floor and the first beam level. The axial force is essentially the same in all uprights, although slightly lower in the left-most upright and third left-most uprights because the bracing support some axial load. Of the six uprights, the bending moment is fractionally higher at node 177 in Element 196. The maximum bending moment in the frame develops at node 13 in Element 12, as shown in Fig. 2.

The axial force and bending moments in the critical uprights between the floor and 1st beam level and in element 13 (here termed Members 1 and 2), as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0049 P*m (Element 196 in LA, 5th upright from the left)
Member 2: N=-3.498P M_{21}=-0.0056 M_{22}=-0.0084 P*m (Element 12 in LA, leftmost upright)

The elastic buckling load of the unbraced frame (P_{cr}), as determined from an LBA analysis, is 22.73kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 89.71kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 22.73kN$

$c_{N1} := 6.000 \qquad N_{cr1} := c_{N1} \cdot P_{cr} \qquad N_{cr1} = 136.38\,kN$

$c_{N2} := 3.498 \qquad N_{cr2} := c_{N2} \cdot P_{cr} \qquad N_{cr2} = 79.51\,kN$

$P_{crb} := 89.71 \cdot kN \qquad N_{crb} := 6 \cdot P_{crb} \qquad N_{crb} = 538.26\,kN$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi \cdot \sqrt{\frac{E \cdot I_u}{N_{crb}}} \qquad L_e = 1.807\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 1.059 \times 10^3\,MPa$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.652$$

$$f_n := if\left(\lambda_c < 1.5, 0.658^{\lambda_c^2} \cdot f_{yu}, \frac{0.977}{\lambda_c^2} \cdot f_{yu}\right) \qquad f_n = 376.649\,MPa$$

Determine the effective area:

Since the section is assumed not to undergo local or distortional buckling, the effective area is taken as the gross area.

$$A_{eu} := A_u \qquad A_{eu} = 508.5\,mm^2$$

Column capacity:

$$N_c := A_{eu} \cdot f_n \qquad N_c = 191.526\,kN$$

Bending capacity of upright

The upright members will not fail by flexural-torsional buckling because of their high torsional rigidity. We therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$M_{su} := f_{yu} \cdot Z_u$ $\quad$ $M_{su} = 6.941\ kN \cdot m$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\ N_c) + C_m M^*/(\phi_b\ M_b \alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.000$, $M_{11}*=0$ and $M_{12}*=c_{M1}*P*m$, $c_{M1}=-0.0052m$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$c_{M1} := 0.0049 \cdot m$

$C_m := 1.0$

$\phi_c := 0.85$ $\quad$ $\phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 22.73\ kN$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\ 2} - 4 \cdot AA_1}\right) \qquad P_1 = 21.054\ kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have N*=c_{N2}*P, c_{N2}=3.498, M_{21}*=c_{M21}*P*m, c_{M21}=0.0056 and M_{22}*=c_{M22}*P*m, c_{M22}=-0.0084; and α_n=1-N*/N_e. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{M21} := 0.0056 \cdot m \qquad c_{M22} := 0.0084 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_c \cdot P_{cr}} \qquad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_c} + \frac{c_{M22} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\ 2} - 4 \cdot AA_2} \right) \qquad P_2 = 21.508\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of Members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 21.054\,kN \qquad P_2 = 21.508\,kN$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 21.054\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level in the GNA analysis.The maximum axial force and bending moment are found in the 4th upright from the left (Element 196). The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_b=M_s) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data :=
GNA - semibraced - PRFSA.xls

Data =

	0	1	2
0	18	0.856	110.361
1	18.5	0.983	113.561
2	19	1.148	116.802
3	19.5	1.369	120.105
4	20	1.685	123.511
5	20.5	2.348	127.478
6	21	3.457	131.81
7	21.25	4.549	134.535
8	21.375	5.589	136.437

9	21.438	6.549	137.869
10	21.469	7.505	139.108
11	21.5	11.68	143.851

$P := \text{for } i \in 0..10$
$\quad ss_i \leftarrow Data_{i,0} \cdot kN$
$\quad ss$

Element 196 (4th upright from the left):

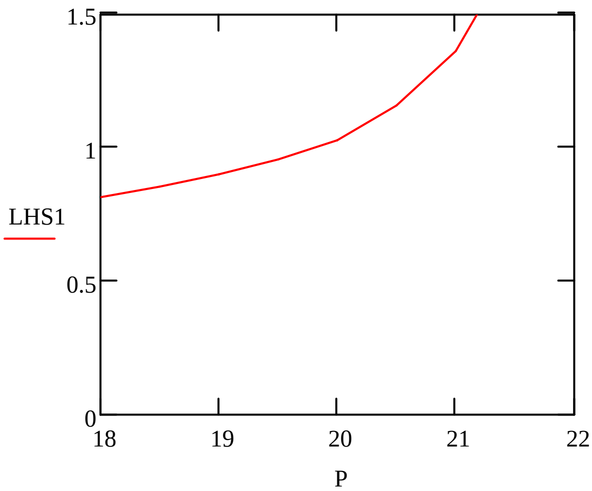
$LHS1 := \text{for } i \in 0..10$
$\quad N \leftarrow Data_{i,2} \cdot kN$
$\quad M \leftarrow Data_{i,1} \cdot kN \cdot m$
$\quad ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_c} + \dfrac{M}{\phi_b \cdot M_{su}}$
$\quad ss$

$P =$ (kN)

	0
0	18
1	18.5
2	19
3	19.5
4	20
5	20.5
6	21
7	21.25
8	21.375
9	21.438
10	21.469

$LHS1 =$

	0
0	0.815
1	0.855
2	0.901
3	0.957
4	1.028
5	1.159
6	1.363
7	1.555
8	1.733
9	1.895
10	2.056



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 3 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS1_{n_u} \qquad y_2 := LHS1_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1$$

$P_u = 19.801\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 19.5 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 17.55\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$$P_{LA} = 21.054\,kN \qquad P_{GNA} = 19.801\,kN \qquad P_{GMNIAc} = 17.55\,kN$$

The factored ultimate load (17.55kN) determined on the basis of a GMNIAc analysis is 19.9% and 12.8% lower than those (21.054kN and 19.801kN) obtained using LA and GNA analyses, respectively.

RF11015

Fully braced rack – Compact cross-section and torsion of uprights ignored

Design Example: Fully braced rack
RF10015 section for uprights and SHS for pallet beams; all members analysed and designed assuming local and distortional buckling does not occur.
Down-aisle displacements only, (2D behaviour).
Flexure only, (while torsion of the uprights will occur in the ultimate limit state, torsion is ignored in the analysis and design calculations).

Kim Rasmussen & Benoit Gilbert

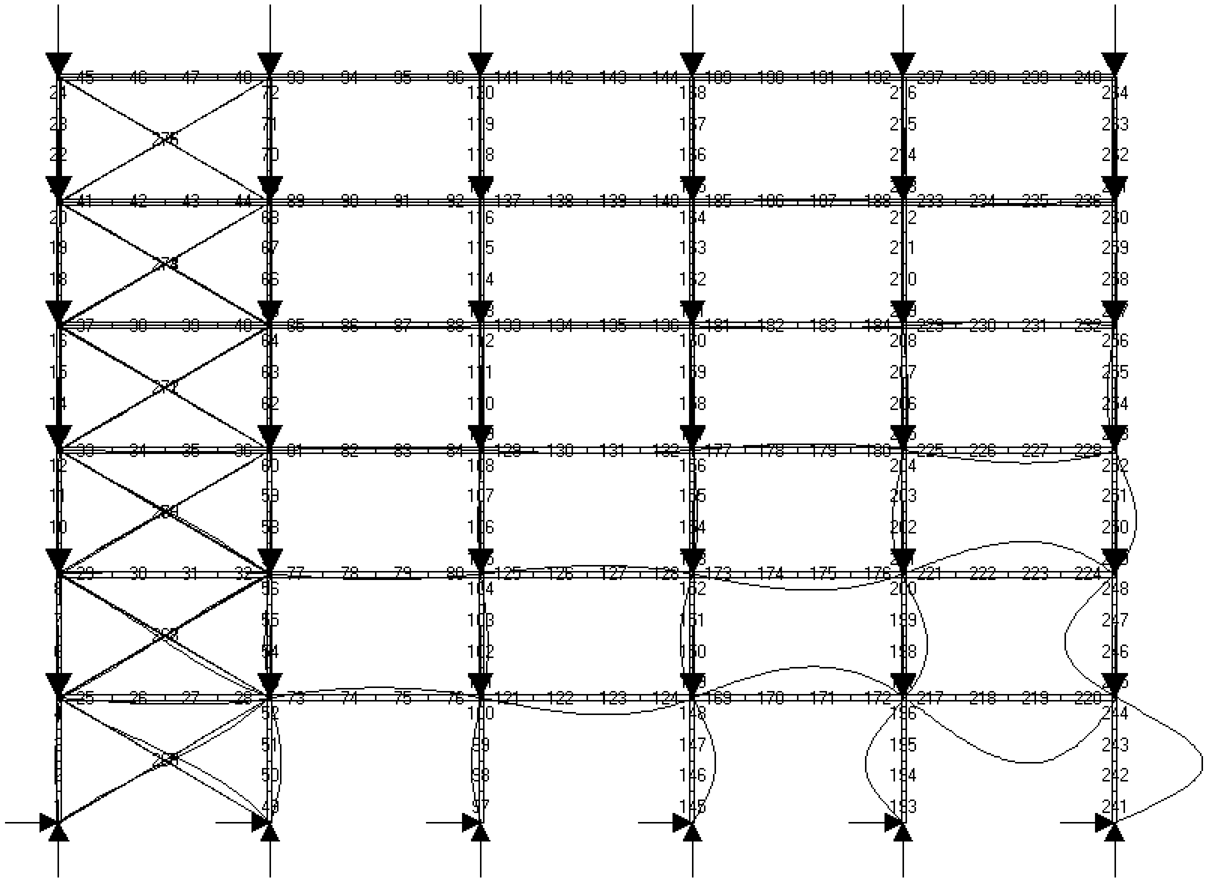


Fig. 1: Fully braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

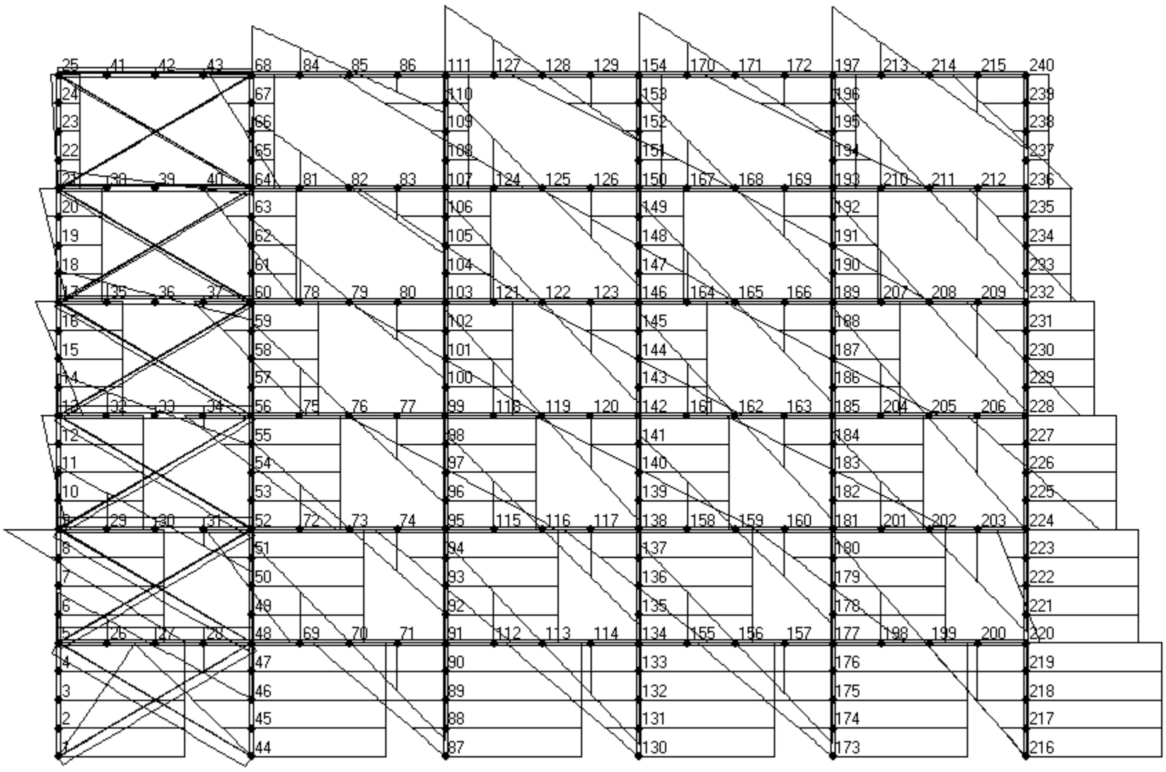


Fig. 2: Node numbers, and axial and bending moment diagrams (LA)

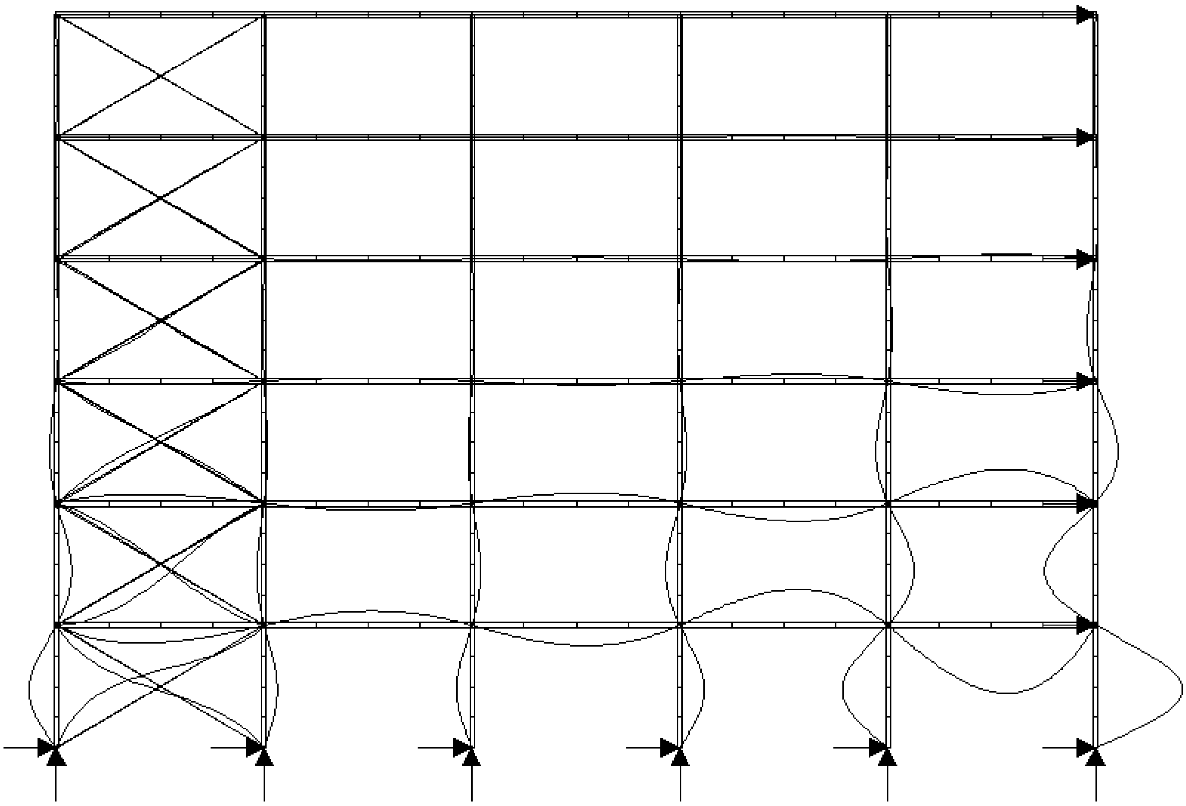


Fig. 3: Buckling mode when all beam levels are restrained (LBA)

Required: The fully braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF100115, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft revised Australian Standard AS4084. The design will be based on LA, GNA and GMNIAc analyses. For design using LA and GNA analyses, member design check is carried out according to AS/NZS4600. The objective of this example is to compared the capacities obtained using these three analysis approaches for a fully braced steel storage rack.

Units:

$m := 1L$ $\quad sec := 1T$ $\quad kg := 1M$ $\quad mm := \frac{m}{1000}$ $\quad N := 1M \cdot 1\frac{L}{1T^2}$ $\quad MPa := \frac{N}{mm^2}$ $\quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

$A_u := 508.5mm^2$ $\quad I_u := 8.484 \cdot 10^5 \cdot mm^4$ $\quad x_{max} := \frac{110}{2} \cdot mm$ $\quad x_{max} = 55\,mm$

$r_u := \sqrt{\frac{I_u}{A_u}}$ $\quad r_u = 40.847\,mm$

$Z_u := \frac{I_u}{x_{max}}$

Beam geometry:

$b_b := 60mm$ $\quad t_b := 4mm$ $\quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2$ $\quad I_b := 4.707 \cdot 10^5 \cdot mm^4$ $\quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}}$ $\quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm$ $\quad t_s := 2mm$

$A_s := 175.9mm^2$ $\quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}}$ $\quad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel tubes):

Upright $f_{yu} := 450MPa$ $\quad$ Beam $f_{yb} := 450MPa$ $\quad$ Brace $f_{ys} := 450MPa$

$E := 210000MPa$ $\quad \nu := 0.3$

1 Design based on LA analysis

The maximum axial force and maximum bending moment develop at node 177 in Element 196 at the first beam level, as shown in Fig. 2.

The axial force and bending moment in the critical upright between the floor and 1st beam level (here termed Member 1), as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0010 P*m (Element 196 in LA)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 99.55kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.56kN. The corresponding buckling mode is shown in Fig. 3. The axial load in the uprights between the floor and the first beam level uprights is found from N_{crb}=6P_{crb} (approximately).

$P_{cr} := 99.55\,kN$

$c_{N1} := 6.000$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 597.3\,kN$

$P_{crb} := 95.56 \cdot kN$ $\quad N_{crb} := 6 \cdot P_{crb}$ $\quad N_{crb} = 573.36\,kN$

Axial capacity of upright Members 1 and 2

As per Clause 4.2.2.1 of the draft Standard, the effective length may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_e := \pi \cdot \sqrt{\frac{E \cdot I_u}{N_{crb}}} \qquad L_e = 1.751\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oc} := \frac{N_{crb}}{A_u} \qquad f_{oc} = 1.128 \times 10^3\,MPa$$

$$\lambda_c := \sqrt{\frac{f_{yu}}{f_{oc}}} \qquad \lambda_c = 0.632$$

$$f_n := if\left(\lambda_c < 1.5, 0.658^{\lambda_c^2} \cdot f_{yu}, \frac{0.977}{\lambda_c^2} \cdot f_{yu}\right) \qquad f_n = 380.774\,MPa$$

Determine the effective area:

Since the section is assumed not to undergo local or distortional buckling, the effective area is taken as the gross area.

$$A_{eu} := A_u \qquad A_{eu} = 508.5\,mm^2$$

Column capacity:

$$N_c := A_{eu} \cdot f_n \qquad N_c = 193.624\,kN$$

Bending capacity of upright

The upright members will not fail by flexural-torsional buckling because of their high torsional rigidity. We therefore only need to check the in-plane capacity.

We ignore local buckling effects and base the section modulus on the full cross-section area:

$M_{su} := f_{yu} \cdot Z_u$ $\qquad$ $M_{su} = 6.941\ kN \cdot m$

Combined compression and bending capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\ N_c) + C_m M^*/(\phi_b\ M_b \alpha) < 1$$

where M* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad \Rightarrow \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the buckling load, as determined from an LBA analysis. It is therefore seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_N{}^*P$, c_N=6.000, $M_{11}{}^*$=0 and $M_{12}{}^*=c_M{}^*P{}^*m$, c_M=-0.0010; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies C_m=0.85.

$c_{M1} := 0.0010 \cdot m$

$C_m := 1.0$

$\phi_c := 0.85$ $\qquad$ $\phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_c \cdot P_{cr}} \qquad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot C_m}{\phi_b \cdot M_{su}} + \frac{1}{P_{cr}} \qquad P_{cr} = 99.55\ kN$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1{}^2 - 4 \cdot AA_1}\right) \qquad P_1 = 27.265\ kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_c} + \frac{c_{M1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{su} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Design capacity of storage rack based on LA analysis:

The maximum factored design load (P) is:

$P_{LA} := P_1$ $\qquad\qquad$ $P_{LA} = 27.265\,\text{kN}$

2 Design based on GNA analysis

The maximum design actions develop near the base of the right-most upright. In the GNA analysis, the axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}=M_s) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data :=
GNA - braced - PRFSA.xls

$$\text{Data} = \begin{pmatrix} 25 & 0.019 & 148.8 & 0.022 & 150 \\ 30 & 0.023 & 178.7 & 0.027 & 180 \\ 35 & 0.029 & 208.5 & 0.036 & 210 \end{pmatrix}$$

$$P := \begin{array}{l} \text{for } i \in 0..2 \\ \quad ss_i \leftarrow \text{Data}_{i,0}\cdot\text{kN} \\ \quad ss \end{array}$$

Element 52 (bottom of 2nd left-most upright):

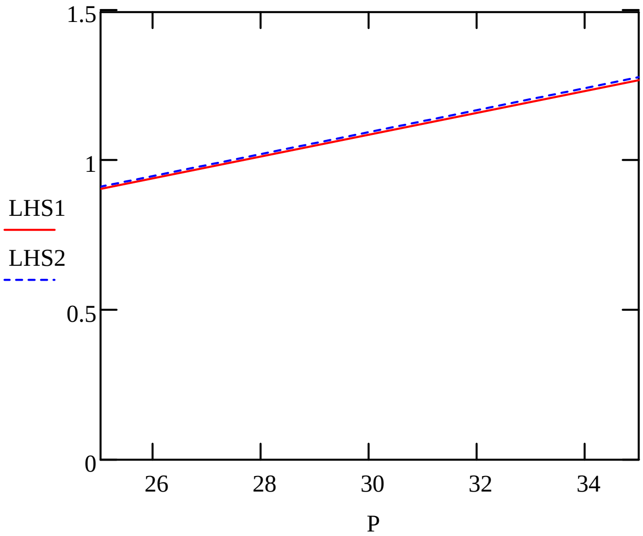
$$LHS1 := \begin{array}{l} \text{for } i \in 0..2 \\ \quad N \leftarrow \text{Data}_{i,2}\cdot\text{kN} \\ \quad M \leftarrow \text{Data}_{i,1}\cdot\text{kN}\cdot\text{m} \\ \quad ss_i \leftarrow \dfrac{N}{\phi_c\cdot N_c} + \dfrac{M}{\phi_b\cdot M_{su}} \\ \quad ss \end{array}$$

$$P = \begin{pmatrix} 25 \\ 30 \\ 35 \end{pmatrix}\text{kN} \qquad LHS1 = \begin{pmatrix} 0.907 \\ 1.09 \\ 1.272 \end{pmatrix}$$

Element 196 (bottom of 2nd right-most upright):

$$LHS2 := \text{for } i \in 0..2 \;\left|\; \begin{aligned} & N \leftarrow Data_{i,4} \cdot kN \\ & M \leftarrow Data_{i,3} \cdot kN \cdot m \\ & ss_i \leftarrow \frac{N}{\phi_c \cdot N_c} + \frac{M}{\phi_b \cdot M_{su}} \\ & ss \end{aligned}\right.$$

$$P = \begin{pmatrix} 25 \\ 30 \\ 35 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.915 \\ 1.098 \\ 1.282 \end{pmatrix}$$



The LHS of the interaction equation varies essentially linearly with the applied load (P) in the load range shown. Determine the value of P producing a LHS of unity by interpolation:

$$n_u := 0 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS2_{n_u} \qquad y_2 := LHS2_{n_u+1}$$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 25\,kN \qquad y_1 = 0.915$$

$$P_u = 27.325\,kN \qquad P_{GNA} := P_u$$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$$P_{max} := 34.0 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 30.6\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$$P_{LA} = 27.265\,kN \qquad P_{GNA} = 27.325\,kN \qquad P_{GMNIAc} = 30.6\,kN$$

The factored ultimate load (30.6kN) determined on the basis of a GMNIAc analysis is 10.9% and 10.7% higher than those (27.265kN and 27.325kN) obtained using LA and GNA analyses, respectively.

RF11015

Unbraced rack – Compact cross-section and torsion of uprights

BD062 Steel Storage Racks

Design Example: Unbraced rack - compact upright cross-section, torsion of uprights

RF10015 section for uprights and SHS for pallet beams.

The uprights and pallet beam members are analysed and designed assuming local and distortional buckling does not occur.

Down-aisle displacements only, (2D behaviour), and torsion. The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

The GMNIAc analysis accounts for warping torsion.

Kim Rasmussen & Benoit Gilbert

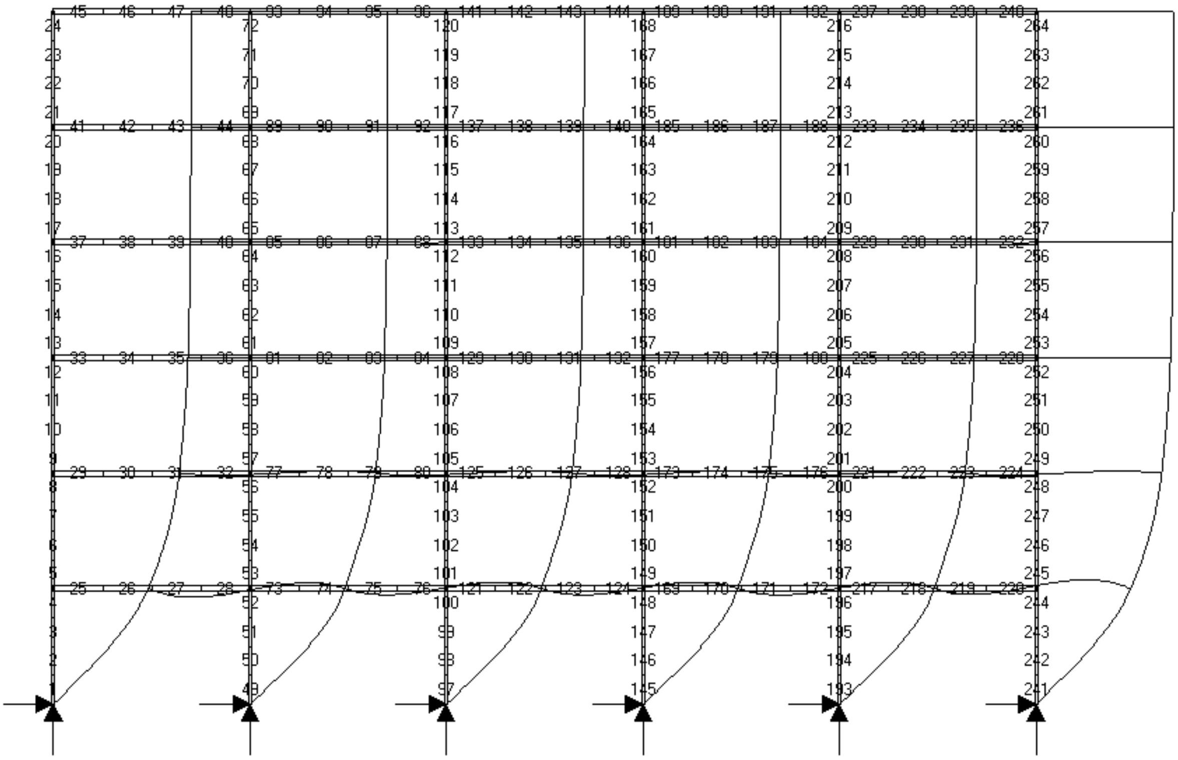


Fig. 1: Unbraced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

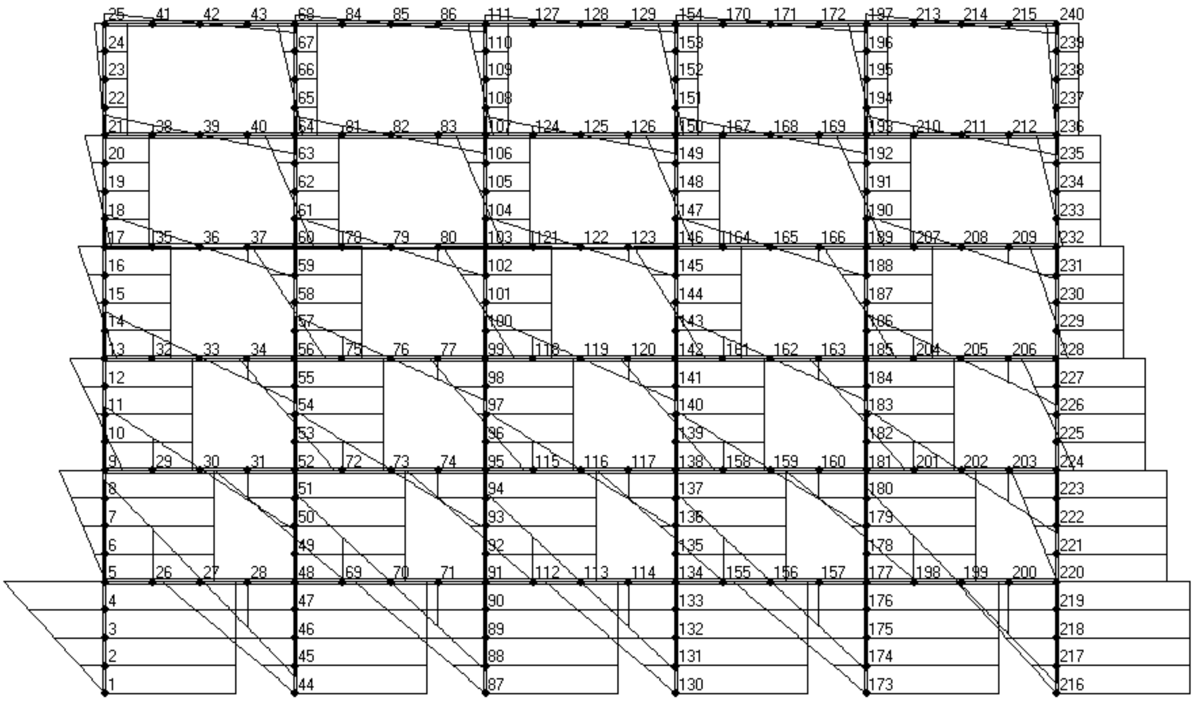


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

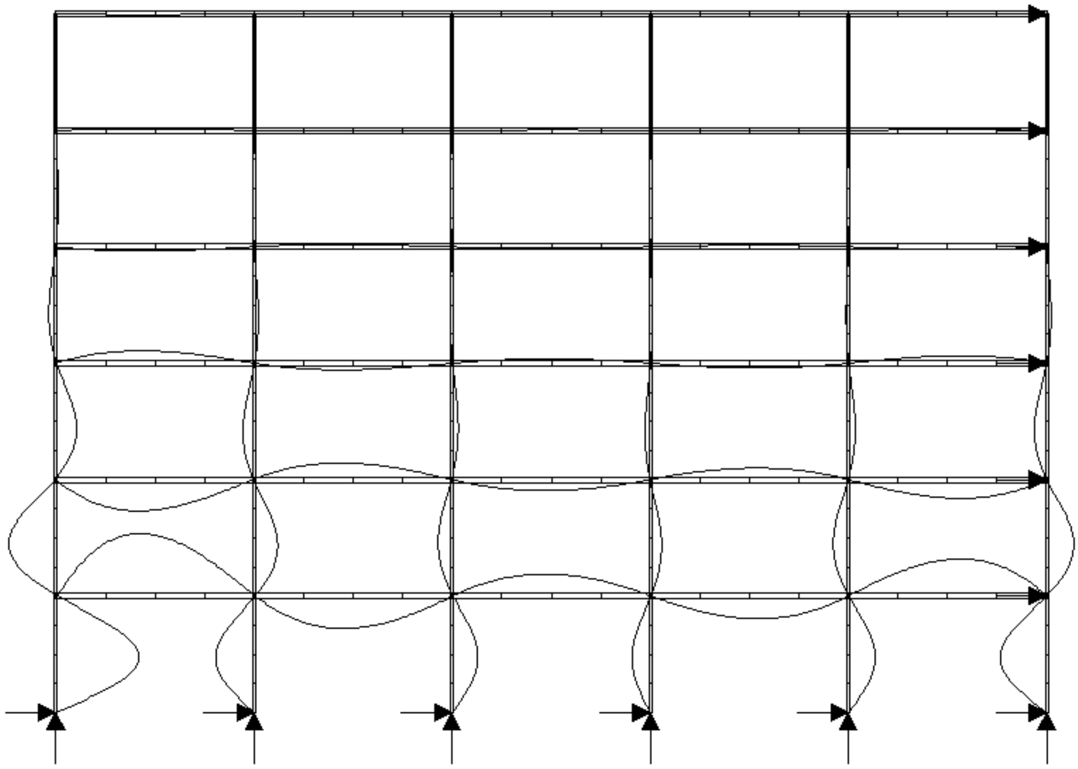


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The unbraced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2

respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAc analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for an unbraced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\dfrac{I_s}{A_s}}$ $\qquad r_s = 9.928\,mm$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa$ $\qquad \nu := 0.3$ $\qquad G := \dfrac{E}{2 \cdot (1 + \nu)}$ $\qquad G = 8.077 \times 10^4\,MPa$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum bending moment develops at node 48 in Element 52 of the 2nd left-most upright at the first beam level, as shown in Fig. 2.The maximum axial force develops in Element 244 of the rightmost upright. The critical member (here termed Member 1) can be shown to be the second left-most upright (containing element 52).

For the uprights between the first and second beam levels, the critical member (Member 2) is the second left-most upright (containing Element 56).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.003P M_{11}= 0 M_{12}=-0.0394 P*m (Element 52 in LA, 2nd upright from left)
Member 2: N=-5.002P M_{21}= 0.0150 M_{22}=-0.0238 P*m (Element 56 in LA, 2nd upright from left)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 11.05kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.43kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from $N_{crb}=c_N P_{crb}$ (approximately).

$P_{cr} := 11.05kN$

$c_{N1} := 6.003$ $\qquad N_{cr1} := c_{N1} \cdot P_{cr}$ $\qquad N_{cr1} = 66.333\,kN$

$c_{N2} := 5.002$ $\qquad N_{cr2} := c_{N2} \cdot P_{cr}$ $\qquad N_{cr2} = 55.272\,kN$

$P_{crb} := 95.43 \cdot kN$ $\qquad N_{crb1} := c_{N1} \cdot P_{crb}$ $\qquad N_{crb1} = 572.866\,kN$

$N_{crb2} := c_{N2} \cdot P_{crb}$ $\qquad N_{crb2} = 477.341\,kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.752\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.127 \times 10^3\ MPa$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 312.158\ MPa$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 312.158\ MPa$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.201$$

$$f_{n1} := \text{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 246.133\ MPa$$

Column capacity:

$$N_{c1} := A_u \cdot f_{n1} \qquad N_{c1} = 125.159\ kN$$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on

N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.919\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$$L_{ez2} := 0.9 \cdot 2 \cdot m \qquad L_{ez2} = 1.8\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 938.723\ MPa$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^2}\right) \qquad f_{oz2} = 238.671\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{\left(f_{oy2} + f_{oz2}\right)^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 202.793\ MPa$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 202.793\ MPa$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.49$$

$$f_{n2} := \mathrm{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^2} \cdot f_{yu}\right) \qquad f_{n2} = 177.768\ MPa$$

Column capacity:

$$N_{c2} := A_u \cdot f_{n2} \qquad N_{c2} = 90.395\ kN$$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Section capacity:

$$M_{suy} := f_{yu} \cdot Z_{uy} \qquad M_{suy} = 6.941\ kN \cdot m$$

Bending capacity (y-axis bending):

$M_{by} := M_{suy}$ $\qquad$ $M_{by} = 6.941\ \text{kN}\cdot\text{m}$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\ N_c) + C_{my}M_y^*/(\phi_b\ M_{by}\alpha_y) < 1$$

where M_y^* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.003$, $M_{11y}^*=0$ and $M_{12y}^*=c_{My}*P*m$, $c_{My1}=-0.0394$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$c_{My1} := 0.0394\cdot m$

$C_m := 1.0$

$\phi_c := 0.85$ $\qquad$ $\phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c\cdot N_{c1}\cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c\cdot N_{c1}} + \frac{c_{My1}\cdot C_m}{\phi_b\cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.153\ \frac{kg}{A^2\cdot m^3\cdot s^4}$$

$$P_1 := \frac{1}{2\cdot AA_1}\cdot\left(BB_1 - \sqrt{BB_1^{\ 2} - 4\cdot AA_1}\right) \qquad P_1 = 9.593\ \text{kN}$$

$$check := \frac{c_{N1}\cdot P_1}{\phi_c\cdot N_{c1}} + \frac{c_{My1}\cdot P_1\cdot C_m}{\phi_b\cdot M_{by}\cdot\left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have $N^*=c_{N2}*P$, $c_{N2}=5.002$, $M_{21y}^*=0.0150*P*m$ and $M_{22y}^*=c_{My}*P*m$, $c_{My2}=-0.0238$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$c_{My2} := 0.0238 \cdot m$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2} \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2}} + \frac{c_{My2} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2} \right) \qquad P_2 = 9.883\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$P_1 = 9.593\,kN$ $\qquad P_2 = 9.883\,kN$

$P_{min} := \min(P_1, P_2)$ $\qquad P_{min} = 9.593\,kN$ $\qquad P_{LA} := P_{min}$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the left side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\ N_c) + M^*_y/\ (\phi_b\ M_{by}) < 1$$

where M^*_y is the maximum bending moment in the member considered.

The (N^*, M^*_y) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data :=
GNA - unbraced - PRFSA.xls

Data =

8	1.231	48.07	0.54	40.02
8.5	1.574	51.08	0.669	42.52
9	2.086	54.1	0.859	45.02
9.5	2.913	57.14	1.159	47.55
10	4.535	60.2	1.734	50.03
10.5	8.985	63.36	3.272	52.52

```
P := for i ∈ 0 .. 5
       ss_i ← Data_{i,0}·kN
       ss
```

Element 52 (2nd left-most upright, between floor and 1st beam level):

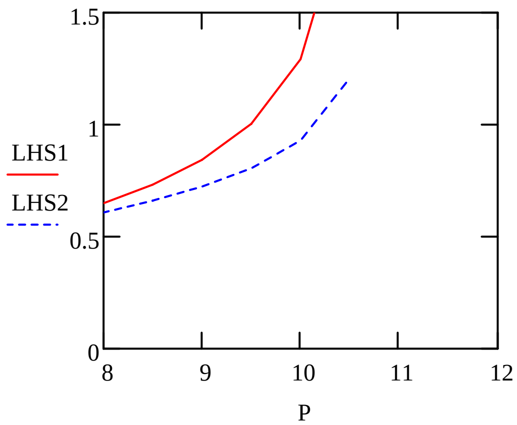
$$LHS1 := \text{for } i \in 0..5 \quad \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c1}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.649 \\ 0.732 \\ 0.842 \\ 1.003 \\ 1.292 \\ 2.034 \end{pmatrix}$$

Element 56 (2nd left-most upright, between 1st and 2nd beam levels):

$$LHS2 := \text{for } i \in 0..5 \quad \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c2}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.607 \\ 0.66 \\ 0.723 \\ 0.804 \\ 0.929 \\ 1.207 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 2 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS1_{n_u} \qquad y_2 := LHS1_{n_u+1}$

$P_u := \dfrac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 9\,kN \qquad y_1 = 0.842$

$P_u = 9.489\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 10.0 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 9\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$P_{LA} = 9.593\ \text{kN}$ $\qquad$ $P_{GNA} = 9.489\ \text{kN}$ $\qquad$ $P_{GMNIAc} = 9\ \text{kN}$

The factored ultimate load (kN) determined on the basis of a GMNIAc analysis is % and % lower than those (9.593kN and 9.489kN) obtained using LA and GNA analyses, respectively.

RF11015

Semi-braced rack – Compact cross-section and torsion of uprights

BD062 Steel Storage Racks

Design Example: Semi-braced rack - compact cross-section, torsion of uprights

RF10015 section for uprights and SHS for pallet beams.

The uprights and pallet beam members are analysed and designed assuming local and distortional buckling does not occur.

Down-aisle displacements only, (2D behaviour), and torsion. The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

The GMNIAc analysis accounts for warping torsion.

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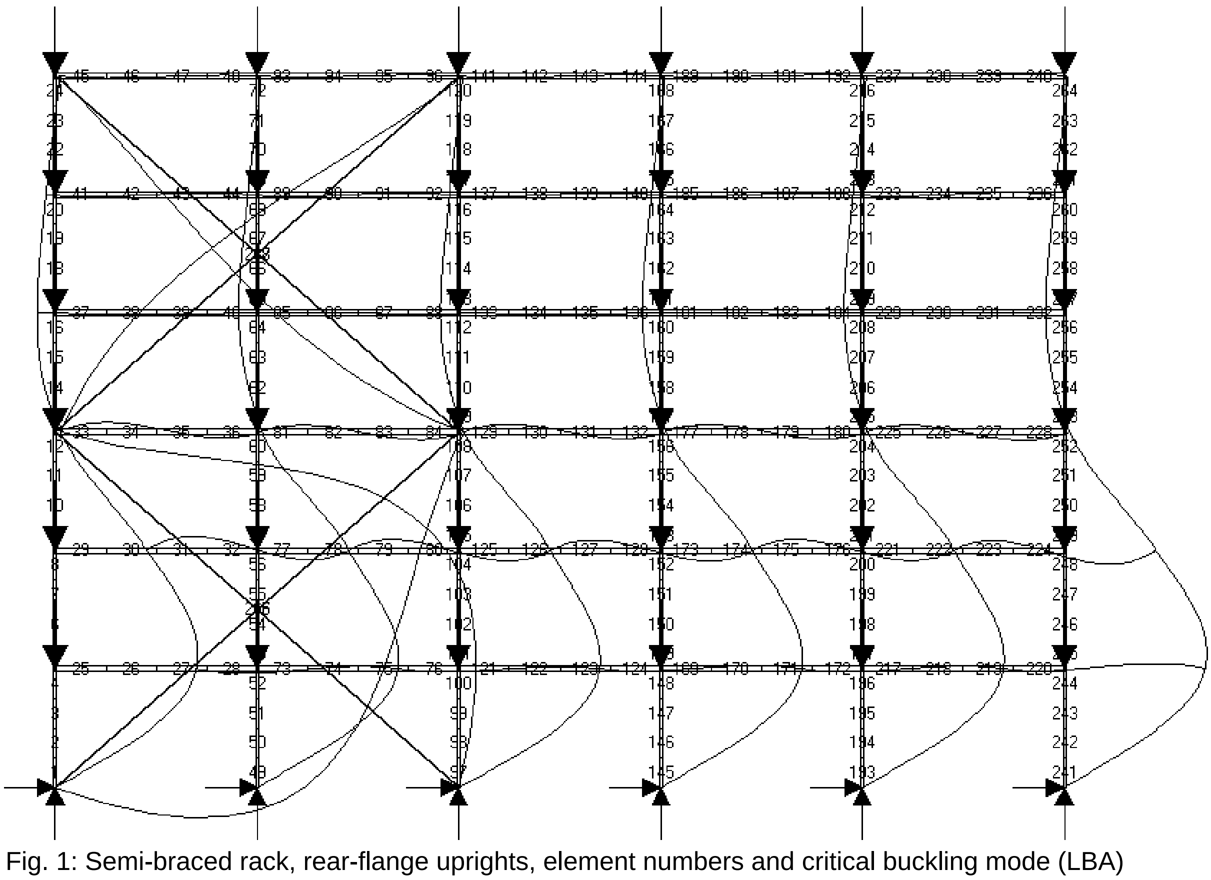


Fig. 1: Semi-braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

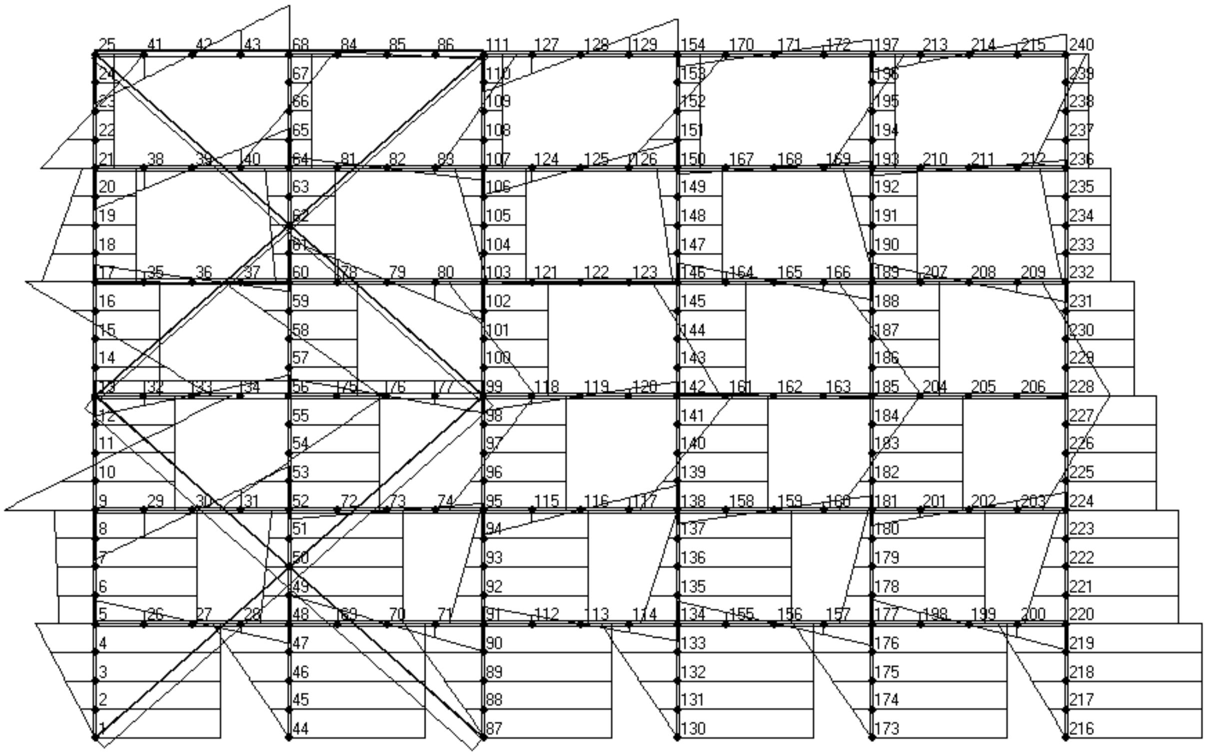


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

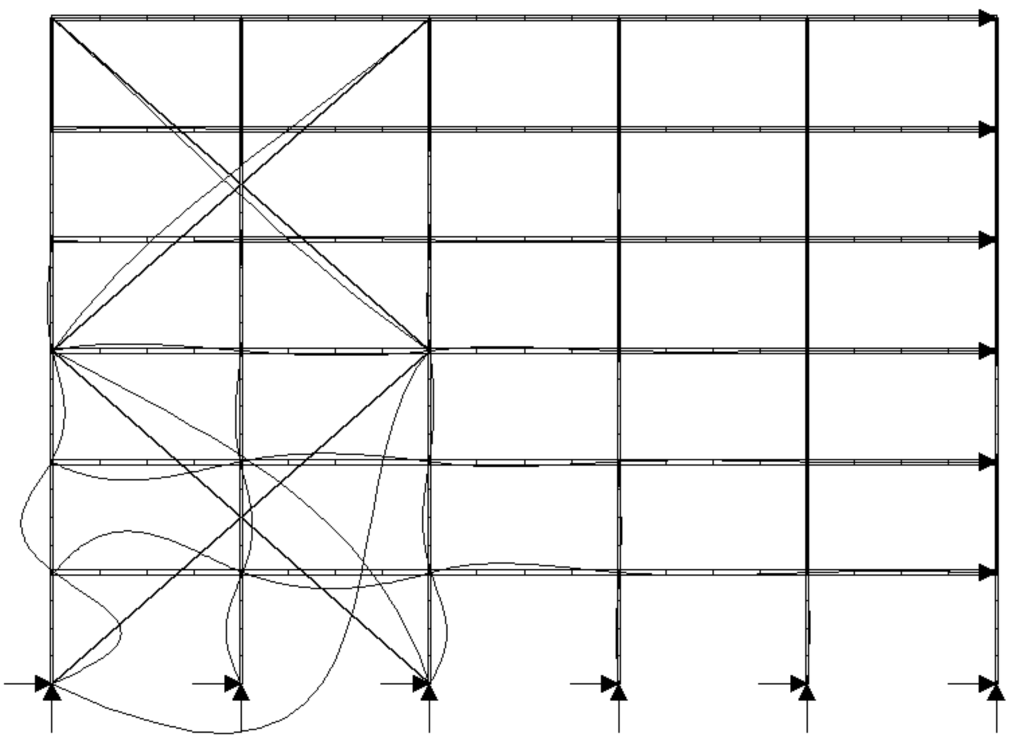


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The semi-braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2

respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAc analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for an semi-braced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$$r_s := \sqrt{\frac{I_s}{A_s}} \qquad r_s = 9.928\,mm$$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$$E := 210000MPa \qquad \nu := 0.3 \qquad G := \frac{E}{2\cdot(1+\nu)} \qquad G = 8.077 \times 10^4\,MPa$$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum axial force and bending moment develop at node 177 in Element 196 of the 2nd right-most upright (here termed Member 1) at the first beam level, as shown in Fig. 2.

For the uprights between the first and second beam levels, the critical member (Member 2) is the second right-most upright (containing Element 197).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0049 P*m (Element 196 in LA, 2nd upright from right)
Member 2: N=-5.000P M_{21}= 0.0005 M_{22}=-0.0020 P*m (Element 197 in LA, 2nd upright from right)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 22.73kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 89.71kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from N_{crb}=$c_N P_{crb}$ (approximately).

$P_{cr} := 22.73kN$

$c_{N1} := 6.000 \qquad N_{cr1} := c_{N1}\cdot P_{cr} \qquad N_{cr1} = 136.38\,kN$

$c_{N2} := 5.000 \qquad N_{cr2} := c_{N2}\cdot P_{cr} \qquad N_{cr2} = 113.65\,kN$

$P_{crb} := 89.71\cdot kN \qquad N_{crb1} := c_{N1}\cdot P_{crb} \qquad N_{crb1} = 538.26\,kN$

$N_{crb2} := c_{N2}\cdot P_{crb} \qquad N_{crb2} = 448.55\,kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on

N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.807\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.059 \times 10^3\ MPa$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 307.892\ MPa$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 307.892\ MPa$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.209$$

$$f_{n1} := \mathrm{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 244.084\ MPa$$

Column capacity:

$$N_{c1} := A_u \cdot f_{n1} \qquad N_{c1} = 124.117\ kN$$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.98\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$L_{ez2} := 0.9 \cdot 2 \cdot m$ $\qquad$ $L_{ez2} = 1.8\ m$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 882.104\ MPa$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^2}\right) \qquad f_{oz2} = 238.671\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{\left(f_{oy2} + f_{oz2}\right)^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 200.712\ MPa$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 200.712\ MPa$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.497$$

$$f_{n2} := \text{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^2} \cdot f_{yu}\right) \qquad f_{n2} = 176.065\ MPa$$

Column capacity:

$N_{c2} := A_u \cdot f_{n2}$ $\qquad$ $N_{c2} = 89.529\ kN$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Section capacity:

$M_{suy} := f_{yu} \cdot Z_{uy}$ $\qquad$ $M_{suy} = 6.941\ kN \cdot m$

Bending capacity (y-axis bending):

$M_{by} := M_{suy}$ $\qquad$ $M_{by} = 6.941\ kN \cdot m$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_{my}M_y^*/(\phi_b\, M_{by}\alpha_y) < 1$$

where M_y* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.000$, $M_{11y}*=0$ and $M_{12y}*=c_{My}*P*m$, $c_{My1}=-0.0049$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{My1} := 0.0049\cdot m$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c\cdot N_{c1}\cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c\cdot N_{c1}} + \frac{c_{My1}\cdot C_m}{\phi_b\cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.102\ \frac{kg}{A^2\cdot m^3\cdot s^4}$$

$$P_1 := \frac{1}{2\cdot AA_1}\cdot\left(BB_1 - \sqrt{BB_1^{\,2} - 4\cdot AA_1}\right) \qquad P_1 = 16.713\,kN$$

$$check := \frac{c_{N1}\cdot P_1}{\phi_c\cdot N_{c1}} + \frac{c_{My1}\cdot P_1\cdot C_m}{\phi_b\cdot M_{by}\cdot\left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have $N^*=c_{N2}*P$, $c_{N2}=5.000$, $M_{21y}*=0.0005*P*m$ and $M_{22y}*=c_{My}*P*m$, $c_{My2}=-0.0020$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{My2} := 0.0020\cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c\cdot N_{c2}\cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c\cdot N_{c2}} + \frac{c_{My2}\cdot C_m}{\phi_b\cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{{BB_2}^2 - 4 \cdot AA_2} \right) \qquad P_2 = 15.005\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 16.713\,kN \qquad P_2 = 15.005\,kN$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 15.005\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the left side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*_y/\,(\phi_b\, M_{by}) < 1$$

where M^*_y is the maximum bending moment in the member considered.

The (N^*, M^*_y) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data := GNA - semibraced - PRFSA.xls

$$Data = \begin{pmatrix} 12 & 0.095 & 72 & 0.051 & 60.01 \\ 14 & 0.153 & 84 & 0.091 & 70.01 \\ 16 & 0.202 & 96.01 & 0.122 & 80.01 \end{pmatrix}$$

```
P := for i ∈ 0 .. 2
       ss_i ← Data_{i,0}·kN
       ss
```

Element 196 (2nd right-most upright, between floor and 1st beam level):

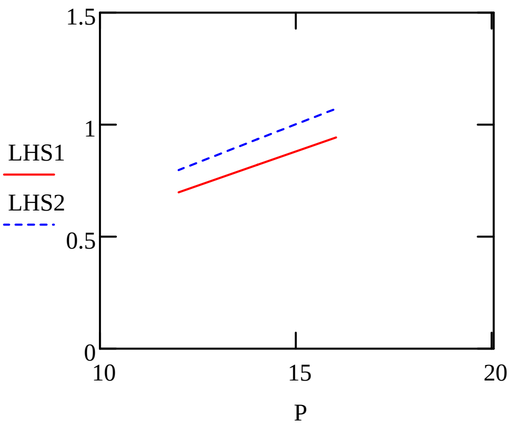
$$LHS1 := \text{for } i \in 0..2 \quad \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c1}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 12 \\ 14 \\ 16 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.698 \\ 0.821 \\ 0.942 \end{pmatrix}$$

Element 197 (2nd right-most upright, between 1st and 2nd beam level)::

$$LHS2 := \text{for } i \in 0..2 \quad \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c2}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 12 \\ 14 \\ 16 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.797 \\ 0.935 \\ 1.071 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$$n_u := 1 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS2_{n_u} \qquad y_2 := LHS2_{n_u+1}$$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 14\,kN \qquad y_1 = 0.935$$

$$P_u = 14.96\,kN \qquad P_{GNA} := P_u$$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$$P_{max} := 18.4 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 16.56\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAc analyses are:

$P_{LA} = 15.005\,\text{kN}$ $\quad$ $P_{GNA} = 14.96\,\text{kN}$ $\quad$ $P_{GMNIAc} = 16.56\,\text{kN}$

The factored ultimate load (16.56kN) determined on the basis of a GMNIAc analysis is 9.39% and 9.66% higher than those (15.005kN and 14.96kN) obtained using LA and GNA analyses, respectively.

RF11015

Fully braced rack – Compact cross-section and torsion of uprights

Appendix 1

BD062 Steel Storage Racks

Design Example: Fully braced rack - compact cross-section, torsion of uprights

RF10015 section for uprights and SHS for pallet beams.

The uprights and pallet beam members are analysed and designed assuming local and distortional buckling does not occur.

Down-aisle displacements only, (2D behaviour), and torsion. The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

The GMNIAc analysis accounts for warping torsion.

Kim Rasmussen & Benoit Gilbert

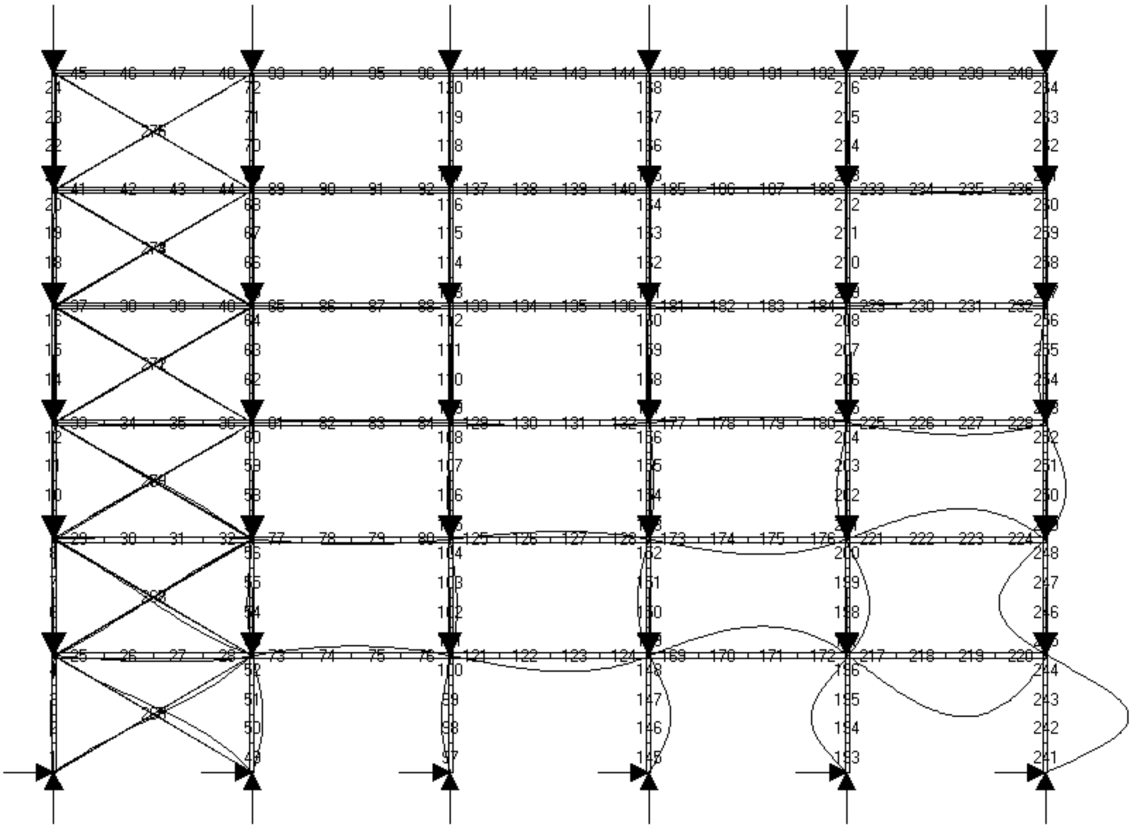


Fig. 1: Fully braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

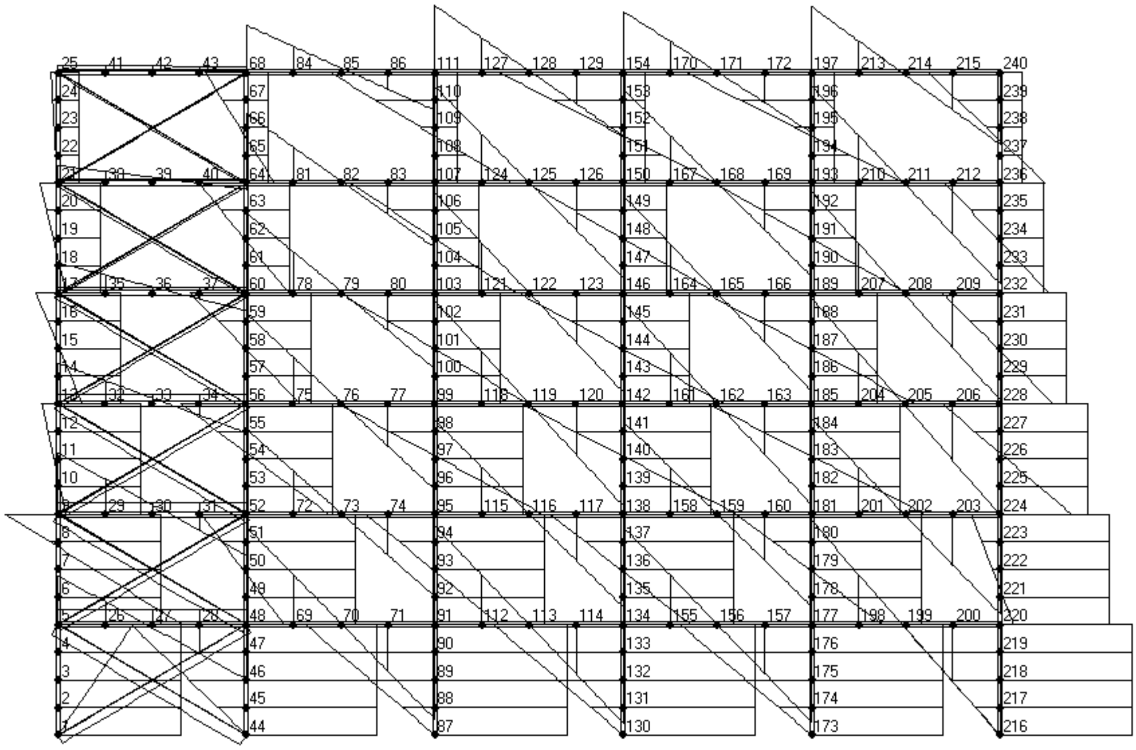


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

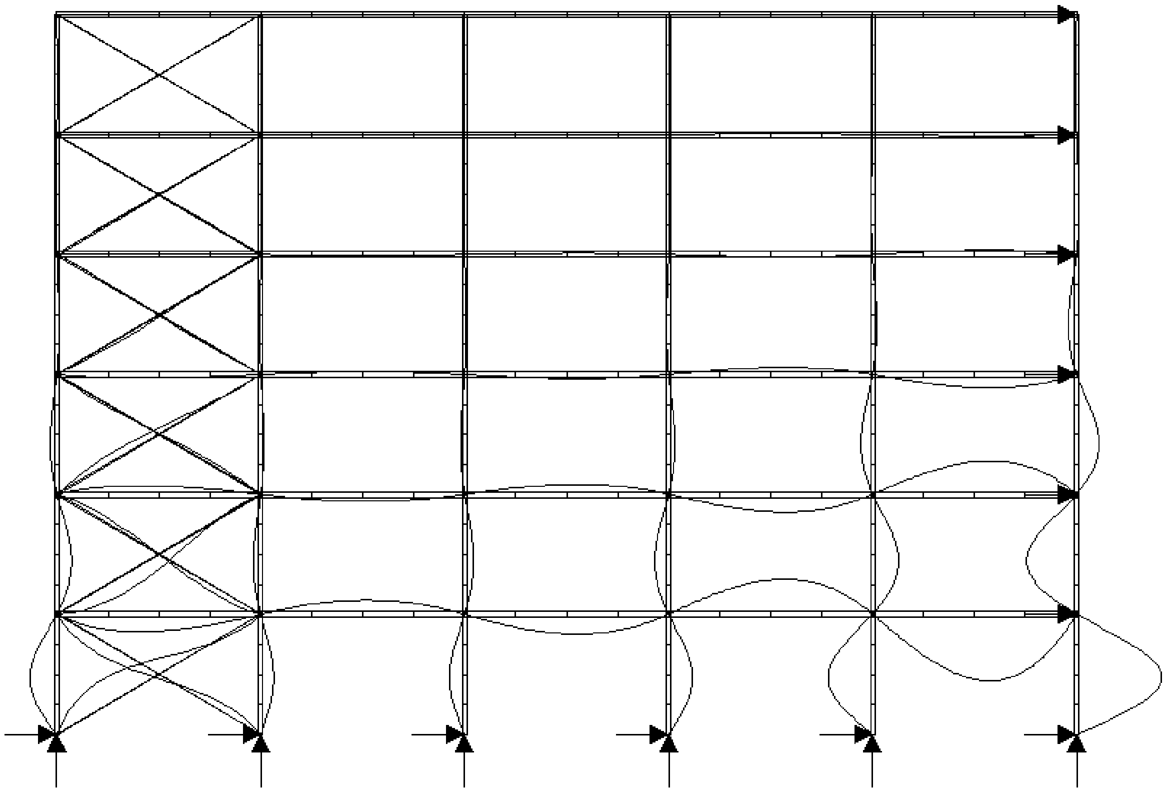


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The fully braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAc analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for a fully braced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1 \frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2$ $I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$r_s := \sqrt{\frac{I_s}{A_s}}$ $r_s = 9.928\ mm$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1 + \nu)}$ $G = 8.077 \times 10^4\ MPa$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum axial force and bending moment develop at node 177 in Element 196 of the 2nd right-most upright (here termed Member 1) at the first beam level, as shown in Fig. 2.

For the uprights between the first and second beam levels, the critical member (Member 2) is the second right-most upright (containing Element 200).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0010 P*m (Element 196 in LA, 2nd upright from right)
Member 2: N=-5.000P M_{21}= 0.0005 M_{22}=-0.0006 P*m (Element 200 in LA, 2nd upright from right)

The elastic buckling load of the braced frame (P_{cr}),as determined from an LBA analysis, is 99.55kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.56kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from $N_{crb}=c_N P_{crb}$ (approximately).

$P_{cr} := 99.55kN$

$c_{N1} := 6.000$ $N_{cr1} := c_{N1} \cdot P_{cr}$ $N_{cr1} = 597.3\ kN$

$c_{N2} := 5.000$ $N_{cr2} := c_{N2} \cdot P_{cr}$ $N_{cr2} = 497.75\ kN$

$P_{crb} := 95.56 \cdot kN$ $N_{crb1} := c_{N1} \cdot P_{crb}$ $N_{crb1} = 573.36\ kN$

$N_{crb2} := c_{N2} \cdot P_{crb}$ $N_{crb2} = 477.8\ kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.751\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.128 \times 10^3\ MPa$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 312.215\ MPa$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 312.215\ MPa$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.201$$

$$f_{n1} := \text{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 246.161\ MPa$$

Column capacity:

$$N_{c1} := A_u \cdot f_{n1} \qquad N_{c1} = 125.173\ kN$$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on

N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.918\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$$L_{ez2} := 0.9 \cdot 2 \cdot m \qquad L_{ez2} = 1.8\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 939.626\ MPa$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^2}\right) \qquad f_{oz2} = 238.671\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{\left(f_{oy2} + f_{oz2}\right)^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 202.824\ MPa$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 202.824\ MPa$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.49$$

$$f_{n2} := \mathrm{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^2} \cdot f_{yu}\right) \qquad f_{n2} = 177.794\ MPa$$

Column capacity:

$$N_{c2} := A_u \cdot f_{n2} \qquad N_{c2} = 90.408\ kN$$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Section capacity:

$$M_{suy} := f_{yu} \cdot Z_{uy} \qquad M_{suy} = 6.941\ kN \cdot m$$

Bending capacity (y-axis bending):

$M_{by} := M_{suy}$ $\qquad M_{by} = 6.941\ kN\cdot m$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_{my} M_y^*/(\phi_b\, M_{by} \alpha_y) < 1$$

where M_y* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}{*}P$, $c_{N1}=6.000$, $M_{11y}{*}=0$ and $M_{12y}{*}=c_{My}{*}P{*}m$, $c_{My1}=-0.0010$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$c_{My1} := 0.0010\cdot m$

$C_m := 1.0$

$\phi_c := 0.85 \qquad \phi_b := 0.9$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1} \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.067\ \frac{kg}{A^2 \cdot m^3 \cdot s^4}$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 17.672\ kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{by} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have $N^*=c_{N2}{*}P$, $c_{N2}=5.000$, $M_{21y}{*}=0.0005{*}P{*}m$ and $M_{22y}{*}=c_{My}{*}P{*}m$, $c_{My2}=-0.0006$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$c_{My2} := 0.0006 \cdot m$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2} \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2}} + \frac{c_{My2} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\ 2} - 4 \cdot AA_2}\right) \qquad P_2 = 15.343\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$P_1 = 17.672\,kN$ $\qquad$ $P_2 = 15.343\,kN$

$P_{min} := \min(P_1, P_2)$ $\qquad$ $P_{min} = 15.343\,kN$ $\qquad$ $P_{LA} := P_{min}$

2 Design based on GNA analysis

The maximum design actions develop near the base of the right-most upright. In the GNA analysis, the axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data := GNA - braced - PRFSA.xls

$$Data = \begin{pmatrix} 10 & 9.4 \times 10^{-3} & 60 & 5.8 \times 10^{-3} & 50 \\ 12.5 & 0.012 & 75 & 7.3 \times 10^{-3} & 62.5 \\ 15 & 0.014 & 90 & 8.9 \times 10^{-3} & 75 \\ 20 & 0.018 & 120 & 0.012 & 100 \end{pmatrix}$$

$P :=$ for $i \in 0..3$

$\quad ss_i \leftarrow Data_{i,0} \cdot kN$

$\quad ss$

Element 196 (2nd right-most upright, between floor and 1st beam level):

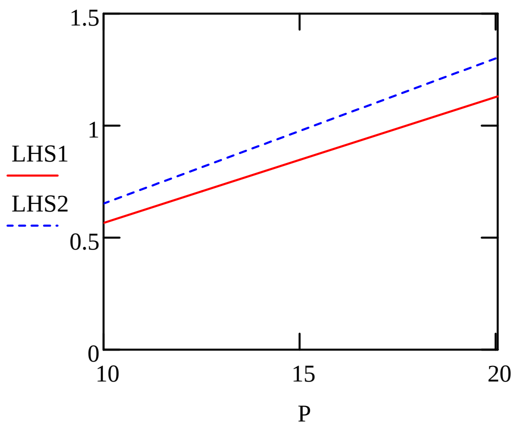
$$LHS1 := \text{for } i \in 0..3 \quad \left| \begin{array}{l} N \leftarrow Data_{i,2} \cdot kN \\ M \leftarrow Data_{i,1} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c1}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 10 \\ 12.5 \\ 15 \\ 20 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.565 \\ 0.707 \\ 0.848 \\ 1.131 \end{pmatrix}$$

Element 200 (2nd right-most upright, between 1st and 2nd beam level)::

$$LHS2 := \text{for } i \in 0..3 \quad \left| \begin{array}{l} N \leftarrow Data_{i,4} \cdot kN \\ M \leftarrow Data_{i,3} \cdot kN \cdot m \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c2}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 10 \\ 12.5 \\ 15 \\ 20 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.652 \\ 0.814 \\ 0.977 \\ 1.303 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 2 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS2_{n_u} \qquad y_2 := LHS2_{n_u+1}$

$P_u := \dfrac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 15\,kN \qquad y_1 = 0.977$

$P_u = 15.347\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAc analysis

The ultimate load (P) obtained directly from a GMNIAc analysis is:

$P_{max} := 21.1 \cdot kN$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$\phi := 0.9$

$P_{GMNIAc} := \phi \cdot P_{max} \qquad P_{GMNIAc} = 18.99\,kN$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAs analyses are:

$P_{LA} = 15.343\,\text{kN}$ $P_{GNA} = 15.347\,\text{kN}$ $P_{GMNIAc} = 18.99\,\text{kN}$

The factored ultimate load (18.99kN) determined on the basis of a GMNIAc analysis is 19.20% and 19.18% higher than those (15.343kN and 15.347kN) obtained using LA and GNA analyses, respectively.

RF11015

Unbraced rack – Non-compact cross-section

BD062 Steel Storage Racks

Design Example: Unbraced rack - non-compact cross-section

RF10015 section for uprights and SHS for pallet beams.

The upright cross-section is prone to local and distortional buckling. Hence, it is analysed using shell elements in the GMNIA analysis and designed accounting for these modes of buckling. The design is based on the Direct Strength Method.

The pallet *beam* members analysed and designed assuming local buckling does not occur.

Down-aisle displacements only, (2D behaviour). The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

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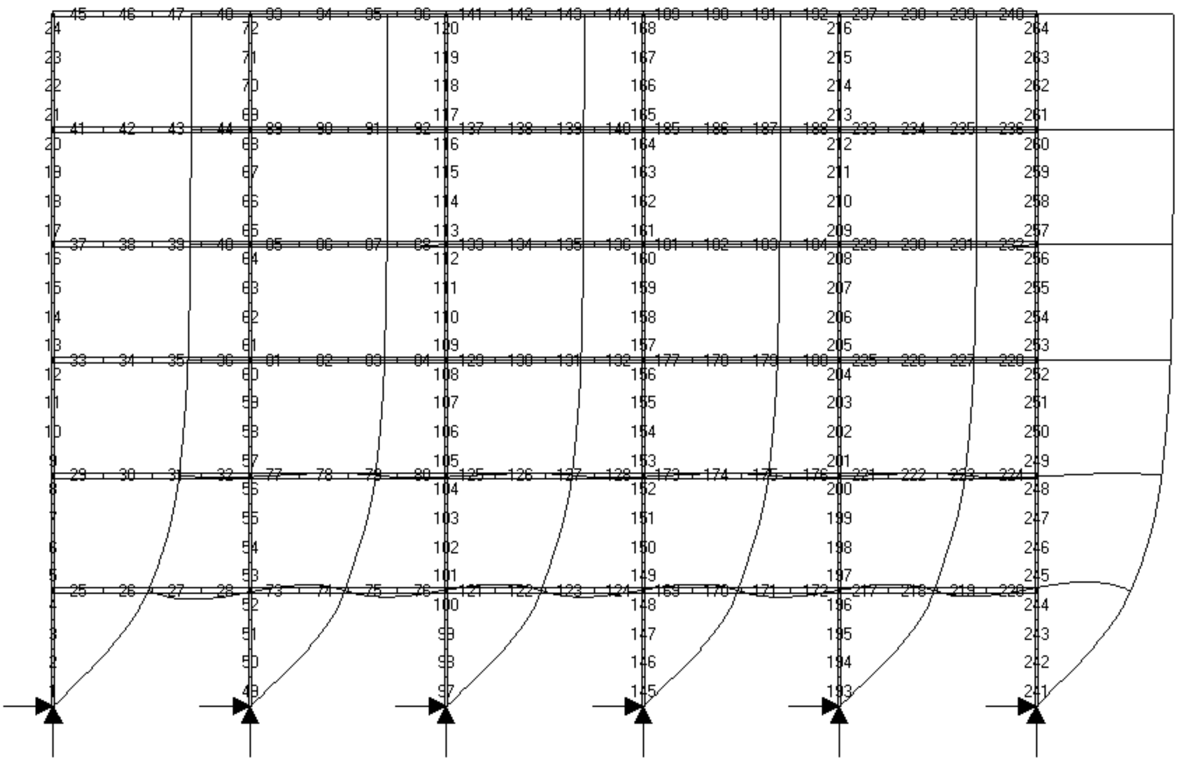


Fig. 1: Unbraced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

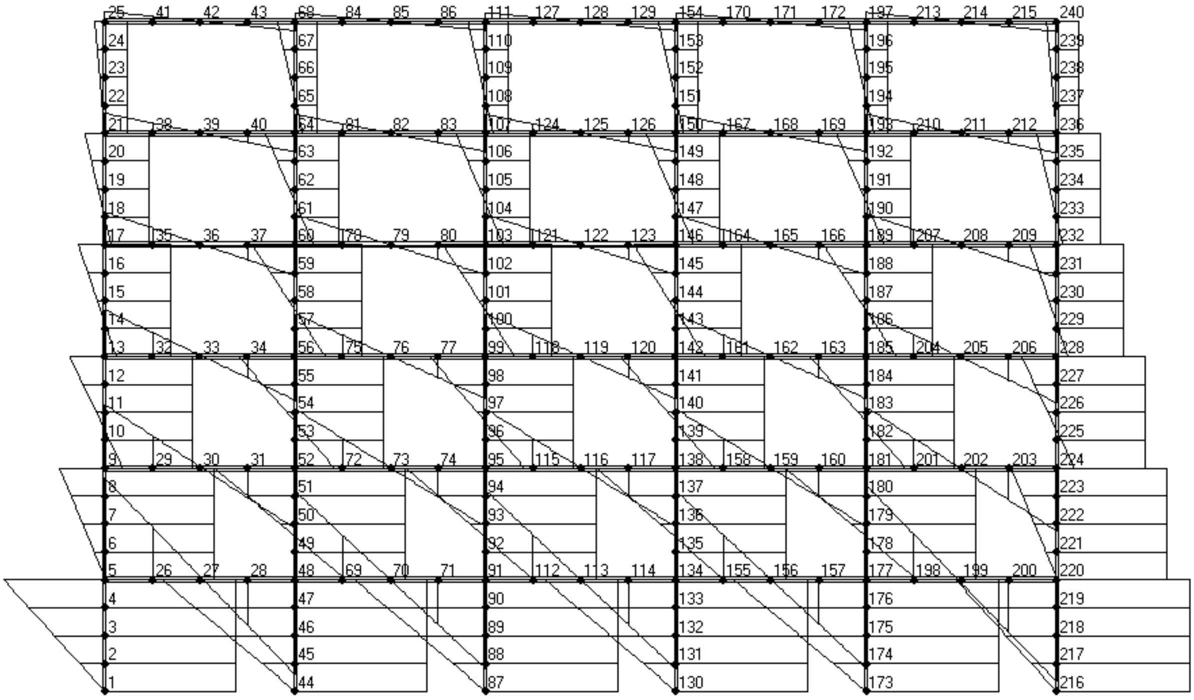


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

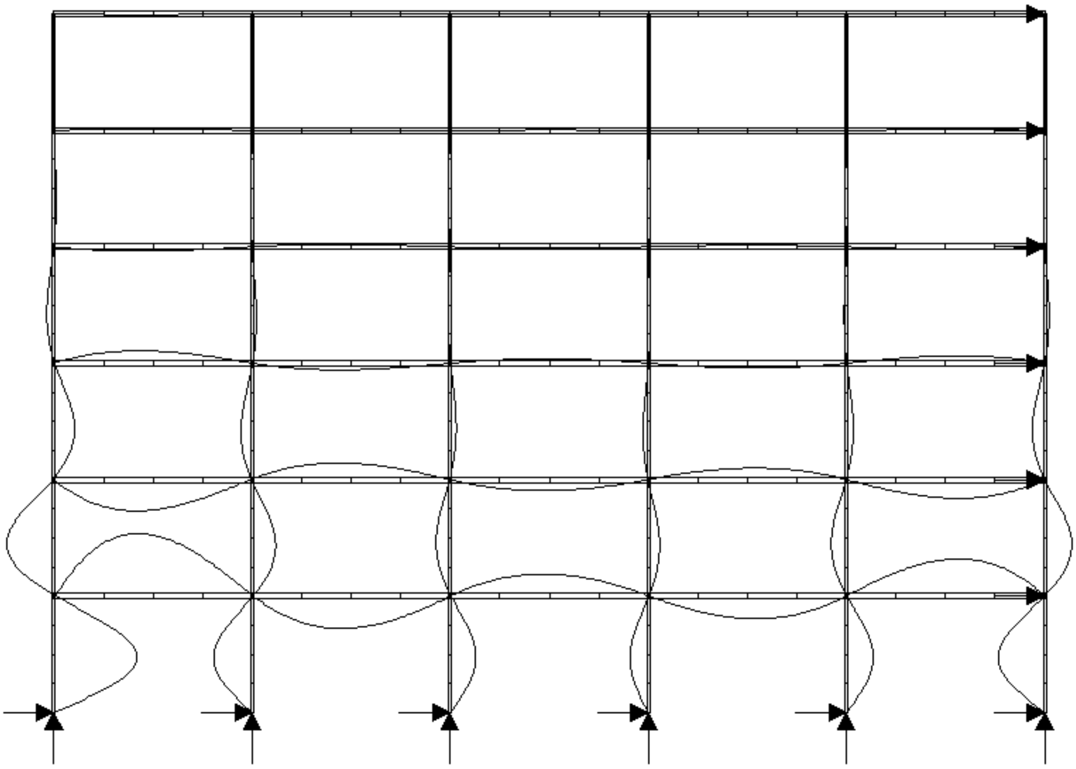


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The unbraced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2

respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAs analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for an unbraced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$$r_s := \sqrt{\frac{I_s}{A_s}} \qquad r_s = 9.928\ mm$$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450MPa$ Beam $f_{yb} := 450MPa$ Brace $f_{ys} := 450MPa$

$E := 210000MPa \qquad \nu := 0.3 \qquad G := \frac{E}{2\cdot(1+\nu)} \qquad G = 8.077 \times 10^4\ MPa$

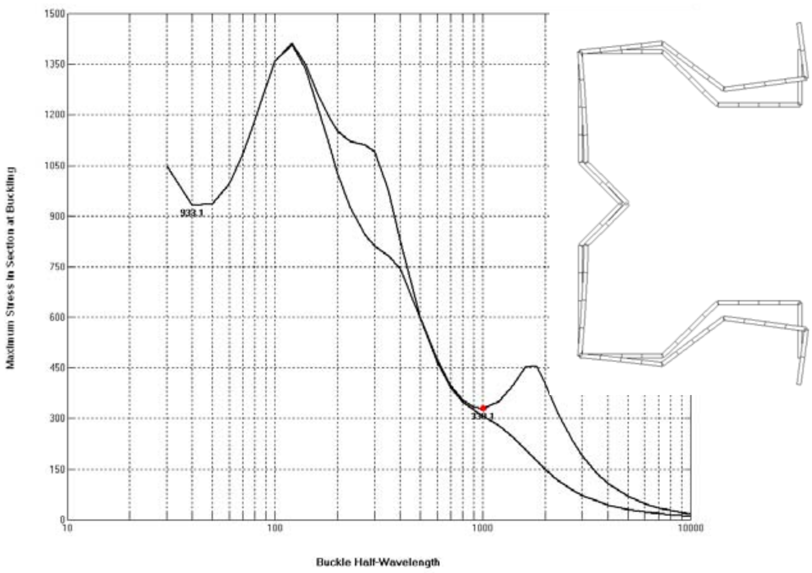


Fig. 4: Buckling stress vs half-wavelength for RF11015 section, axial compression, 1st and 2nd mode of buckling

Thinwall has been used to determined the local and distortional buckling stresses for axial compresion and bending about the x- and y-axes. The buckling stress versus buckle half-wavelength is shown in Fig. 4 for axial compression. The distortional buckling minimum is found (as the second mode of buckling) at a half-wavelength of 1000 mm.
The symmetry axis is the y-axis.

$f_{ol} := 933\cdot MPa \qquad f_{od} := 330\cdot MPa$

$f_{olx} := 1035\cdot MPa \qquad f_{odx} := 450\cdot MPa$

$f_{oly} := 946\cdot MPa \qquad f_{ody} := 449\cdot MPa$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum bending moment develops at node 48 in Element 52 of the 2nd left-most upright at the first beam level, as shown in Fig. 2.The maximum axial force develops in Element 244 of the rightmost upright. The critical member (here termed Member 1) can be shown to be the second left-most upright (containing element 52).

For the uprights between the first and second beam levels, the critical member (Member 2) is the second left-most upright (containing Element 56).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.003P M_{11}= 0 M_{12}=-0.0394 P*m (Element 52 in LA, 2nd upright from left)
Member 2: N=-5.002P M_{21}= 0.0150 M_{22}=-0.0238 P*m (Element 56 in LA, 2nd upright from left)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 11.05kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.43kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from $N_{crb}=c_N P_{crb}$ (approximately).

$P_{cr} := 11.05 kN$

$c_{N1} := 6.003$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 66.333 \, kN$

$c_{N2} := 5.002$ $\quad N_{cr2} := c_{N2} \cdot P_{cr}$ $\quad N_{cr2} = 55.272 \, kN$

$P_{crb} := 95.43 \cdot kN$ $\quad N_{crb1} := c_{N1} \cdot P_{crb}$ $\quad N_{crb1} = 572.866 \, kN$

$N_{crb2} := c_{N2} \cdot P_{crb}$ $\quad N_{crb2} = 477.341 \, kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.752 \ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4 \ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.127 \times 10^3\ MPa$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 312.158\ MPa$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 312.158\ MPa$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.201$$

$$f_{n1} := \mathrm{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 246.133\ MPa$$

Determine columns capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce1} := A_u \cdot f_{n1} \qquad N_{ce1} = 125.159\ kN$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43\ kN$$

$$\lambda_l := \sqrt{\frac{N_{ce1}}{N_{ol}}} \qquad \lambda_l = 0.514$$

$$N_{cl} := \mathrm{if}\left[\lambda_l < 0.776, N_{ce1}, \left[1 - 0.15 \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4}\right] \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4} \cdot N_{ce1}\right] \qquad N_{cl} = 125.159\ kN$$

Distortional buckling:

$$N_{yu} := A_u \cdot f_{yu} \qquad N_{yu} = 228.825\ kN$$

$$N_{od} := A_u \cdot f_{od} \qquad N_{od} = 167.805\ kN$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \mathrm{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25 \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right] \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6} \cdot N_{yu}\right] \qquad N_{cd} = 150.542\ kN$$

Column capacity:

$N_{c1} := \min(N_{ce1}, N_{cl}, N_{cd})$ $\qquad N_{c1} = 125.159\ kN$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.919\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$$L_{ez2} := 0.9 \cdot 2 \cdot m \qquad L_{ez2} = 1.8\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 938.723\ MPa$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^{\ 2}} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^{\ 2}}\right) \qquad f_{oz2} = 238.671\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{(f_{oy2} + f_{oz2})^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 202.793\ MPa$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 202.793\ MPa$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.49$$

$$f_{n2} := \text{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^{\ 2}} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^{\ 2}} \cdot f_{yu}\right) \qquad f_{n2} = 177.768\ MPa$$

Determine column capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce2} := A_u \cdot f_{n2} \qquad N_{ce2} = 90.395\ kN$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43\ kN$$

$$\lambda_l := \sqrt{\frac{N_{ce2}}{N_{ol}}} \qquad \lambda_l = 0.437$$

$$N_{cl} := \text{if}\left[\lambda_l < 0.776, N_{ce2}, \left[1 - 0.15\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\right]\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\cdot N_{ce2}\right] \qquad N_{cl} = 90.395\ \text{kN}$$

Distortional buckling:

$$N_{yu} := A_u \cdot f_{yu} \qquad N_{yu} = 228.825\ \text{kN}$$

$$N_{od} := A_u \cdot f_{od} \qquad N_{od} = 167.805\ \text{kN}$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \text{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right]\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\cdot N_{yu}\right] \qquad N_{cd} = 150.542\ \text{kN}$$

Column capacity:

$$N_{c2} := \min(N_{ce2}, N_{cl}, N_{cd}) \qquad N_{c2} = 90.395\ \text{kN}$$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Since the cross-section is slender, local and distortional buckling need to be accounted for. This is achieved using the Direct Strength Method.

Section capacity:

$$M_{suy} := f_{yu} \cdot Z_{uy} \qquad M_{suy} = 6.941\ \text{kN}\cdot\text{m}$$

Overall buckling:

$$M_{bey} := M_{suy} \qquad M_{bey} = 6.941\ \text{kN}\cdot\text{m}$$

Local buckling:

$$M_{oly} := Z_{uy} \cdot f_{oly} \qquad M_{oly} = 14.592\ \text{m kN}$$

$$\lambda_{ly} := \sqrt{\frac{M_{bey}}{M_{oly}}} \qquad \lambda_{ly} = 0.69$$

$$M_{bly} := \text{if}\left[\lambda_{ly} < 0.776, M_{bey}, \left[1 - 0.15\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\right]\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\cdot M_{bey}\right] \qquad M_{bly} = 6.941\ \text{kN}\cdot\text{m}$$

Distortional buckling:

$$M_{yuy} := Z_{uy} \cdot f_{yu} \qquad M_{yuy} = 6.941\ \text{kN}\cdot\text{m}$$

$$M_{ody} := Z_{uy} \cdot f_{ody} \qquad M_{ody} = 6.926\, kN \cdot m$$

$$\lambda_{dy} := \sqrt{\frac{M_{yuy}}{M_{ody}}} \qquad \lambda_{dy} = 1.001$$

$$M_{bdy} := \text{if}\left[\lambda_{dy} < 0.673, M_{yuy}, \left[1 - 0.22 \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5}\right] \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5} \cdot M_{yuy}\right] \qquad M_{bdy} = 5.41\, m\, kN$$

Bending capacity (y-axis bending):

$$M_{by} := \min\left(M_{bey}, M_{bly}, M_{bdy}\right) \qquad M_{by} = 5.41\, kN \cdot m$$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\, N_c) + C_{my} M_y^*/(\phi_b\, M_{by} \alpha_y) < 1$$

where M_y* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.003$, $M_{11y}*=0$ and $M_{12y}*=c_{My}*P*m$, $c_{My1}=-0.0394$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{My1} := 0.0394 \cdot m$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1} \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.155\, \frac{kg}{A^2 \cdot m^3 \cdot s^4}$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 9.3\, kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{by} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have $N^*=c_{N2}*P$, $c_{N2}=5.002$, $M_{21y}*=0.0150*P*m$ and $M_{22y}*=c_{My}*P*m$, $c_{My2}=-0.0238$; and $\alpha_n=1-N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{My2} := 0.0238 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2} \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2}} + \frac{c_{My2} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2}\right) \qquad P_2 = 9.649\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 9.3\,kN \qquad P_2 = 9.649\,kN$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 9.3\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the left side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*_y/\, (\phi_b\, M_{by}) < 1$$

where M^*_y is the maximum bending moment in the member considered.

The (N^*, M^*_y) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

$$\text{Data} := \text{GNA - unbraced - PRFSA.xls}$$

$$\text{Data} = \begin{pmatrix} 8 & 1.231 & 48.07 & 0.54 & 40.02 \\ 8.5 & 1.574 & 51.08 & 0.669 & 42.52 \\ 9 & 2.086 & 54.1 & 0.859 & 45.02 \\ 9.5 & 2.913 & 57.14 & 1.159 & 47.55 \\ 10 & 4.535 & 60.2 & 1.734 & 50.03 \\ 10.5 & 8.985 & 63.36 & 3.272 & 52.52 \end{pmatrix}$$

$$P := \text{for } i \in 0..5 \quad \left| \begin{array}{l} ss_i \leftarrow \text{Data}_{i,0} \cdot \text{kN} \\ ss \end{array} \right.$$

Element 52 (2nd left-most upright, between floor and 1st beam level):

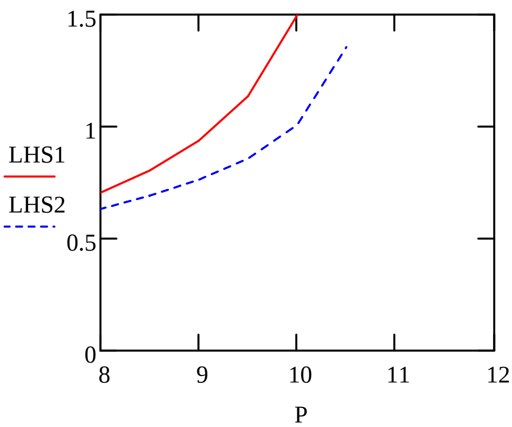
$$\text{LHS1} := \text{for } i \in 0..5 \quad \left| \begin{array}{l} N \leftarrow \text{Data}_{i,2} \cdot \text{kN} \\ M \leftarrow \text{Data}_{i,1} \cdot \text{kN} \cdot \text{m} \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c1}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} \text{kN} \qquad \text{LHS1} = \begin{pmatrix} 0.705 \\ 0.803 \\ 0.937 \\ 1.135 \\ 1.497 \\ 2.441 \end{pmatrix}$$

Element 56 (2nd left-most upright, between 1st and 2nd beam levels):

$$\text{LHS2} := \text{for } i \in 0..5 \quad \left| \begin{array}{l} N \leftarrow \text{Data}_{i,4} \cdot \text{kN} \\ M \leftarrow \text{Data}_{i,3} \cdot \text{kN} \cdot \text{m} \\ ss_i \leftarrow \dfrac{N}{\phi_c \cdot N_{c2}} + \dfrac{M}{\phi_b \cdot M_{by}} \\ ss \end{array} \right.$$

$$P = \begin{pmatrix} 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \end{pmatrix} \text{kN} \qquad \text{LHS2} = \begin{pmatrix} 0.632 \\ 0.691 \\ 0.762 \\ 0.857 \\ 1.007 \\ 1.356 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 2 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := \text{LHS1}_{n_u} \qquad y_2 := \text{LHS1}_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 9\,\text{kN} \qquad y_1 = 0.937$$

$P_u = 9.159\,\text{kN} \qquad P_{GNA} := P_u$

3 Design based on GMNIAs analysis

The ultimate load (P) obtained directly from a GMNIAs analysis is:

$$P_{max} := 8.1 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAs} := \phi \cdot P_{max} \qquad P_{GMNIAs} = 7.29\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAs analyses are:

$$P_{LA} = 9.3\,kN \qquad P_{GNA} = 9.159\,kN \qquad P_{GMNIAs} = 7.29\,kN$$

The factored ultimate load (7.29kN) determined on the basis of a GMNIAs analysis is 27.6% and 25.6% lower than those (9.3kN and 9.159kN) obtained using LA and GNA analyses, respectively.

RF11015

Semi-braced rack – Non-compact cross-section

BD062 Steel Storage Racks

Design Example: Semi-braced rack - non-compact cross-section

RF10015 section for uprights and SHS for pallet beams.

The upright cross-section is prone to local and distortional buckling. Hence, it is analysed using shell elements in the GMNIA analysis and designed accounting for these modes of buckling. The design is based on the Direct Strength Method.

The pallet *beam* members analysed and designed assuming local buckling does not occur.

Down-aisle displacements only, (2D behaviour). The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

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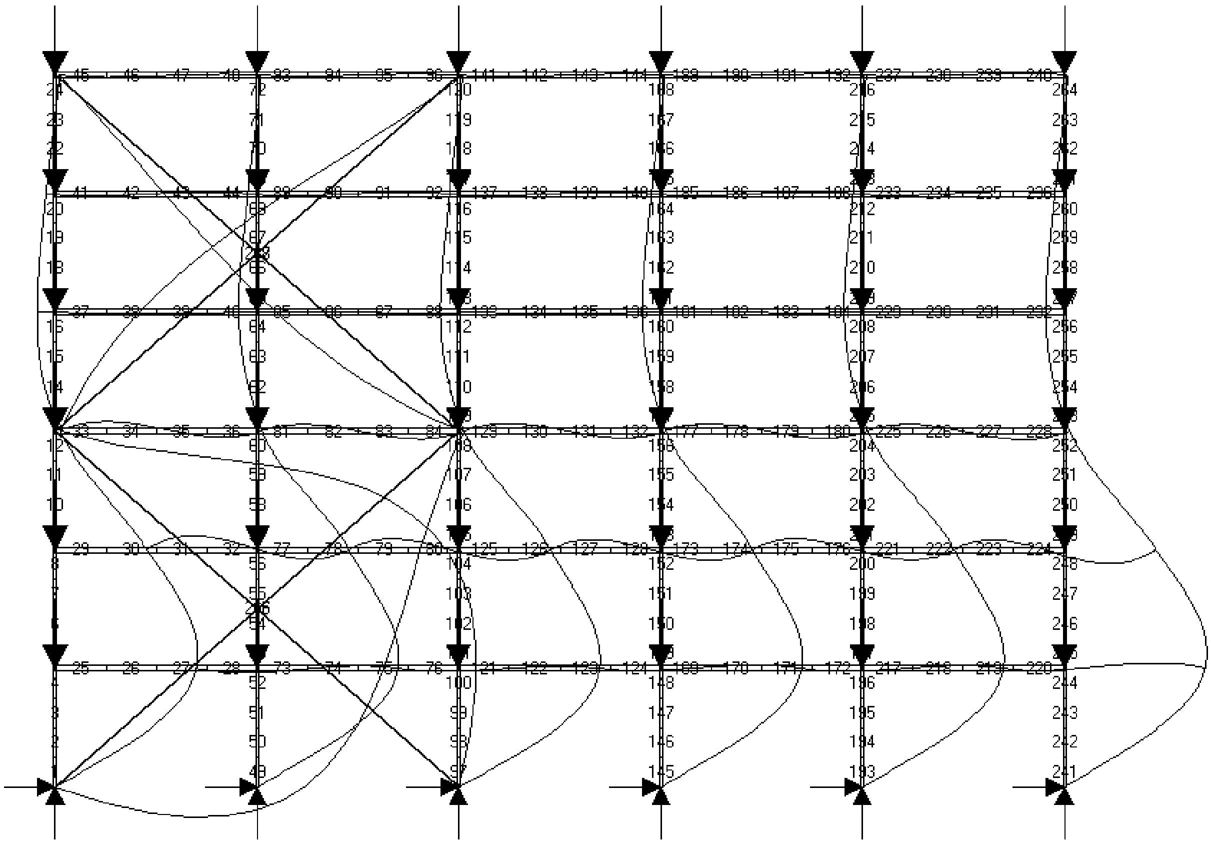


Fig. 1: Semi-braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

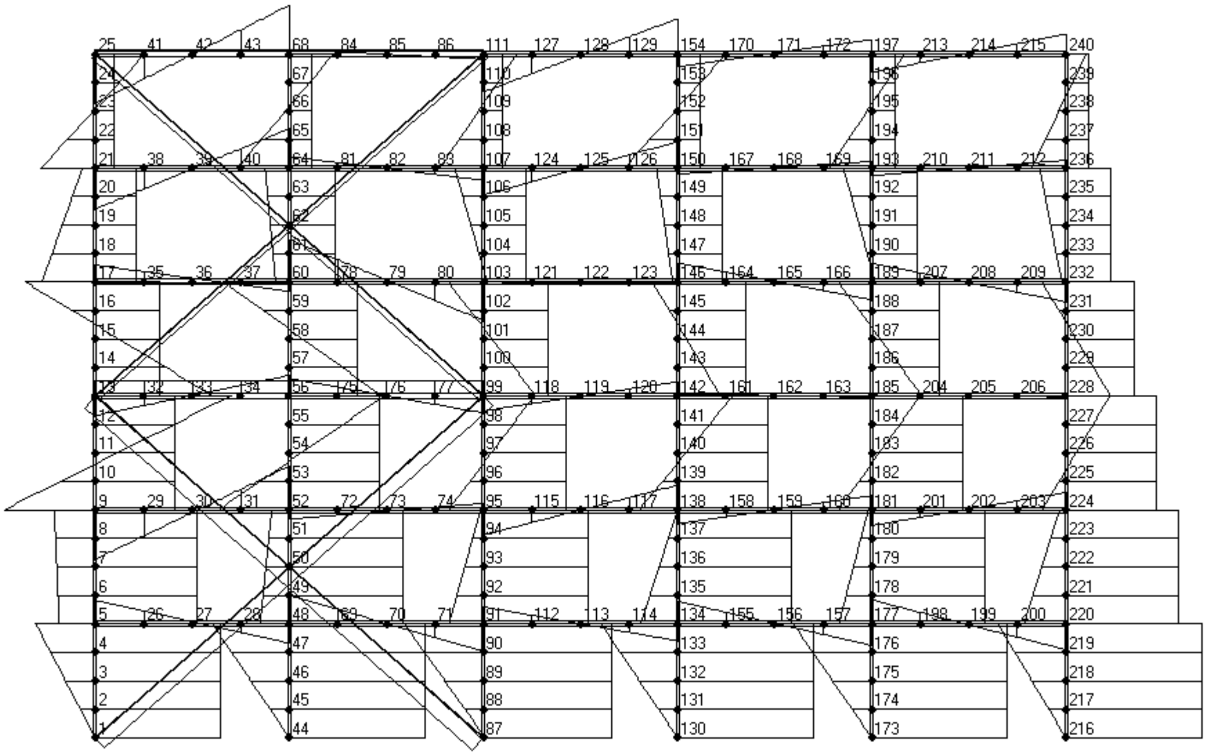


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

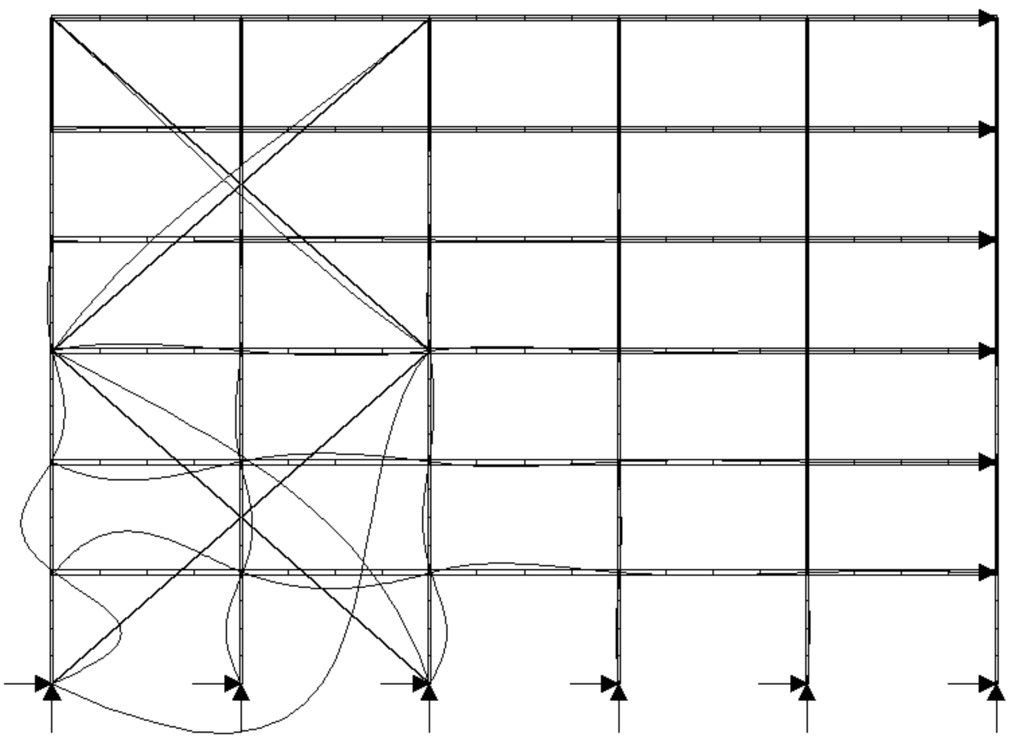


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

Required: The semi-braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2

respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAs analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for an semi-braced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

Upright geometry:

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9mm^2 \quad I_s := 1.733605 \cdot 10^4 \cdot mm^4$

$$r_s := \sqrt{\frac{I_s}{A_s}} \qquad r_s = 9.928\ \text{mm}$$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450\text{MPa}$ Beam $f_{yb} := 450\text{MPa}$ Brace $f_{ys} := 450\text{MPa}$

$$E := 210000\text{MPa} \qquad \nu := 0.3 \qquad G := \frac{E}{2\cdot(1+\nu)} \qquad G = 8.077 \times 10^4\ \text{MPa}$$

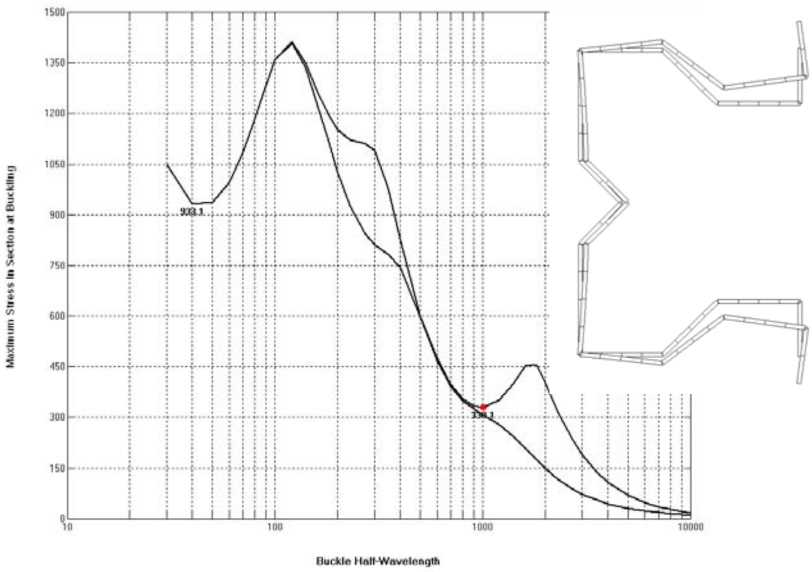


Fig. 4: Buckling stress vs half-wavelength for RF11015 section, axial compression, 1st and 2nd mode of buckling

Thinwall has been used to determined the local and distortional buckling stresses for axial compresion and bending about the x- and y-axes. The buckling stress versus buckle half-wavelength is shown in Fig. 4 for axial compression. The distortional buckling minimum is found (as the second mode of buckling) at a half-wavelength of 1000 mm.
The symmetry axis is the y-axis.

$$f_{ol} := 933\cdot\text{MPa} \qquad f_{od} := 330\cdot\text{MPa}$$

$$f_{olx} := 1035\cdot\text{MPa} \qquad f_{odx} := 450\cdot\text{MPa}$$

$$f_{oly} := 946\cdot\text{MPa} \qquad f_{ody} := 449\cdot\text{MPa}$$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum axial force and bending moment develop at node 177 in Element 196 of the 2nd right-most upright (here termed Member 1) at the first beam level, as shown in Fig. 2.

For the uprights between the first and second beam levels, the critical member (Member 2) is the second right-most upright (containing Element 197).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0049 P*m (Element 196 in LA, 2nd upright from right)
Member 2: N=-5.000P M_{21}= 0.0005 M_{22}=-0.0020 P*m (Element 197 in LA, 2nd upright from right)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 22.73kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 89.71kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from $N_{crb}=c_N P_{crb}$ (approximately).

$P_{cr} := 22.73kN$

$c_{N1} := 6.000$ $N_{cr1} := c_{N1} \cdot P_{cr}$ $N_{cr1} = 136.38\,kN$

$c_{N2} := 5.000$ $N_{cr2} := c_{N2} \cdot P_{cr}$ $N_{cr2} = 113.65\,kN$

$P_{crb} := 89.71 \cdot kN$ $N_{crb1} := c_{N1} \cdot P_{crb}$ $N_{crb1} = 538.26\,kN$

$N_{crb2} := c_{N2} \cdot P_{crb}$ $N_{crb2} = 448.55\,kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.807\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.059 \times 10^3\,\text{MPa}$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975\,\text{MPa}$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 307.892\,\text{MPa}$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 307.892\,\text{MPa}$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.209$$

$$f_{n1} := \text{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 244.084\,\text{MPa}$$

Determine columns capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce1} := A_u \cdot f_{n1} \qquad N_{ce1} = 124.117\,\text{kN}$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43\,\text{kN}$$

$$\lambda_l := \sqrt{\frac{N_{ce1}}{N_{ol}}} \qquad \lambda_l = 0.511$$

$$N_{cl} := \text{if}\left[\lambda_l < 0.776, N_{ce1}, \left[1 - 0.15 \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4}\right] \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4} \cdot N_{ce1}\right] \qquad N_{cl} = 124.117\,\text{kN}$$

Distortional buckling:

$$N_{yu} := A_u \cdot f_{yu} \qquad N_{yu} = 228.825\,\text{kN}$$

$$N_{od} := A_u \cdot f_{od} \qquad N_{od} = 167.805\,\text{kN}$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \text{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25 \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right] \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6} \cdot N_{yu}\right] \qquad N_{cd} = 150.542\,\text{kN}$$

Column capacity:

$$N_{c1} := \min(N_{ce1}, N_{cl}, N_{cd}) \qquad N_{c1} = 124.117\ \text{kN}$$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.98\ \text{m}$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$$L_{ez2} := 0.9 \cdot 2 \cdot m \qquad L_{ez2} = 1.8\ \text{m}$$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 882.104\ \text{MPa}$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^{\ 2}} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^{\ 2}}\right) \qquad f_{oz2} = 238.671\ \text{MPa}$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{(f_{oy2} + f_{oz2})^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 200.712\ \text{MPa}$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 200.712\ \text{MPa}$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.497$$

$$f_{n2} := \text{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^{\ 2}} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^{\ 2}} \cdot f_{yu}\right) \qquad f_{n2} = 176.065\ \text{MPa}$$

Determine column capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce2} := A_u \cdot f_{n2} \qquad N_{ce2} = 89.529\ \text{kN}$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43\ \text{kN}$$

$$\lambda_l := \sqrt{\frac{N_{ce2}}{N_{ol}}} \qquad \lambda_l = 0.434$$

$$N_{cl} := \text{if}\left[\lambda_l < 0.776, N_{ce2}, \left[1 - 0.15\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\right]\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\cdot N_{ce2}\right] \qquad N_{cl} = 89.529\,kN$$

Distortional buckling:

$$N_{yu} := A_u\cdot f_{yu} \qquad N_{yu} = 228.825\,kN$$

$$N_{od} := A_u\cdot f_{od} \qquad N_{od} = 167.805\,kN$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \text{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right]\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\cdot N_{yu}\right] \qquad N_{cd} = 150.542\,kN$$

Column capacity:

$$N_{c2} := \min\left(N_{ce2}, N_{cl}, N_{cd}\right) \qquad N_{c2} = 89.529\,kN$$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Since the cross-section is slender, local and distortional buckling need to be accounted for. This is achieved using the Direct Strength Method.

Section capacity:

$$M_{suy} := f_{yu}\cdot Z_{uy} \qquad M_{suy} = 6.941\,kN\cdot m$$

Overall buckling:

$$M_{bey} := M_{suy} \qquad M_{bey} = 6.941\,kN\cdot m$$

Local buckling:

$$M_{oly} := Z_{uy}\cdot f_{oly} \qquad M_{oly} = 14.592\,m\,kN$$

$$\lambda_{ly} := \sqrt{\frac{M_{bey}}{M_{oly}}} \qquad \lambda_{ly} = 0.69$$

$$M_{bly} := \text{if}\left[\lambda_{ly} < 0.776, M_{bey}, \left[1 - 0.15\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\right]\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\cdot M_{bey}\right] \qquad M_{bly} = 6.941\,kN\cdot m$$

Distortional buckling:

$$M_{yuy} := Z_{uy}\cdot f_{yu} \qquad M_{yuy} = 6.941\,kN\cdot m$$

$$M_{ody} := Z_{uy} \cdot f_{ody} \qquad M_{ody} = 6.926\,kN \cdot m$$

$$\lambda_{dy} := \sqrt{\frac{M_{yuy}}{M_{ody}}} \qquad \lambda_{dy} = 1.001$$

$$M_{bdy} := if\left[\lambda_{dy} < 0.673, M_{yuy}, \left[1 - 0.22 \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5}\right] \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5} \cdot M_{yuy}\right] \qquad M_{bdy} = 5.41\ m\,kN$$

Bending capacity (y-axis bending):

$$M_{by} := \min\left(M_{bey}, M_{bly}, M_{bdy}\right) \qquad M_{by} = 5.41\,kN \cdot m$$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$$N^*/(\phi_c\ N_c) + C_{my} M_y^*/(\phi_b\ M_{by} \alpha_y) < 1$$

where M_y* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$$\alpha = 1\text{-}N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e\text{-}N^*)$$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e\text{-}N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.000$, $M_{11y}*=0$ and $M_{12y}*=c_{My}*P*m$, $c_{My1}=-0.0049$; and $\alpha_n=1\text{-}N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies C_m=0.85.

$$c_{My1} := 0.0049 \cdot m$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1} \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.102\ \frac{kg}{A^2 \cdot m^3 \cdot s^4}$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 16.515\,kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{by} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have N*=c_{N2}*P, c_{N2}=5.000, M_{21y}*=0.0005*P*m and M_{22y}*=c_{My}*P*m, c_{My2}=-0.0020; and α_n=1-N*/N_e. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{My2} := 0.0020 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2} \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2}} + \frac{c_{My2} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2}\right) \qquad P_2 = 14.947\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 16.515\,kN \qquad P_2 = 14.947\,kN$$

$$P_{min} := \min(P_1, P_2) \qquad P_{min} = 14.947\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

The maximum design actions develop at the first beam level.The maximum axial force is found in the rightmost upright, while the maximum moment is found in the second upright from the left side where the horizontal force is acting. The axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*_y/\,(\phi_b\, M_{by}) < 1$$

where M^*_y is the maximum bending moment in the member considered.

The (N*, M^*_y) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data :=

GNA - semibraced - PRFSA.xls

$$\text{Data} = \begin{pmatrix} 12 & 0.095 & 72 & 0.051 & 60.01 \\ 14 & 0.153 & 84 & 0.091 & 70.01 \\ 16 & 0.202 & 96.01 & 0.122 & 80.01 \end{pmatrix}$$

$$P := \text{for } i \in 0..2 \;\Big|\; ss_i \leftarrow Data_{i,0} \cdot kN \;;\; ss$$

Element 196 (2nd right-most upright, between floor and 1st beam level):

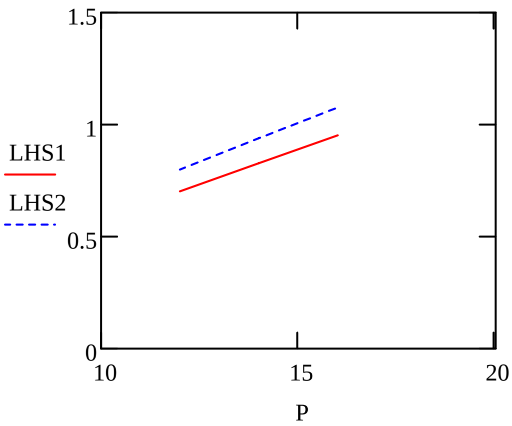
$$LHS1 := \text{for } i \in 0..2 \;\Big|\; N \leftarrow Data_{i,2} \cdot kN \;;\; M \leftarrow Data_{i,1} \cdot kN \cdot m \;;\; ss_i \leftarrow \frac{N}{\phi_c \cdot N_{c1}} + \frac{M}{\phi_b \cdot M_{by}} \;;\; ss$$

$$P = \begin{pmatrix} 12 \\ 14 \\ 16 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.702 \\ 0.828 \\ 0.952 \end{pmatrix}$$

Element 197 (2nd right-most upright, between 1st and 2nd beam level)::

$$LHS2 := \text{for } i \in 0..2 \;\Big|\; N \leftarrow Data_{i,4} \cdot kN \;;\; M \leftarrow Data_{i,3} \cdot kN \cdot m \;;\; ss_i \leftarrow \frac{N}{\phi_c \cdot N_{c2}} + \frac{M}{\phi_b \cdot M_{by}} \;;\; ss$$

$$P = \begin{pmatrix} 12 \\ 14 \\ 16 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.799 \\ 0.939 \\ 1.076 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 1 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS2_{n_u} \qquad y_2 := LHS2_{n_u+1}$

$$P_u := \frac{1 - y_1}{y_2 - y_1} \cdot (x_2 - x_1) + x_1 \qquad x_1 = 14\,kN \qquad y_1 = 0.939$$

$P_u = 14.891\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAs analysis

The ultimate load (P) obtained directly from a GMNIAs analysis is:

$$P_{max} := 16.4 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAs} := \phi \cdot P_{max} \qquad P_{GMNIAs} = 14.76\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAs analyses are:

$$P_{LA} = 14.947\,kN \qquad P_{GNA} = 14.891\,kN \qquad P_{GMNIAs} = 14.76\,kN$$

The factored ultimate load (14.76kN) determined on the basis of a GMNIAs analysis is 1.2% and 0.9% lower than those (14.947kN and 14.891kN) obtained using LA and GNA analyses, respectively.

RF11015

Fully braced rack – Non-compact cross-section

BD062 Steel Storage Racks

Design Example: Fully braced rack - non-compact cross-section

RF10015 section for uprights and SHS for pallet beams.

The upright cross-section is prone to local and distortional buckling. Hence, it is analysed using shell elements in the GMNIA analysis and designed accounting for these modes of buckling. The design is based on the Direct Strength Method.

The pallet *beam* members analysed and designed assuming local buckling does not occur.

Down-aisle displacements only, (2D behaviour). The uprights are restrained in the cross-aisle direction, thus failure occurs by flexure in the down-aisle direction and torsion.

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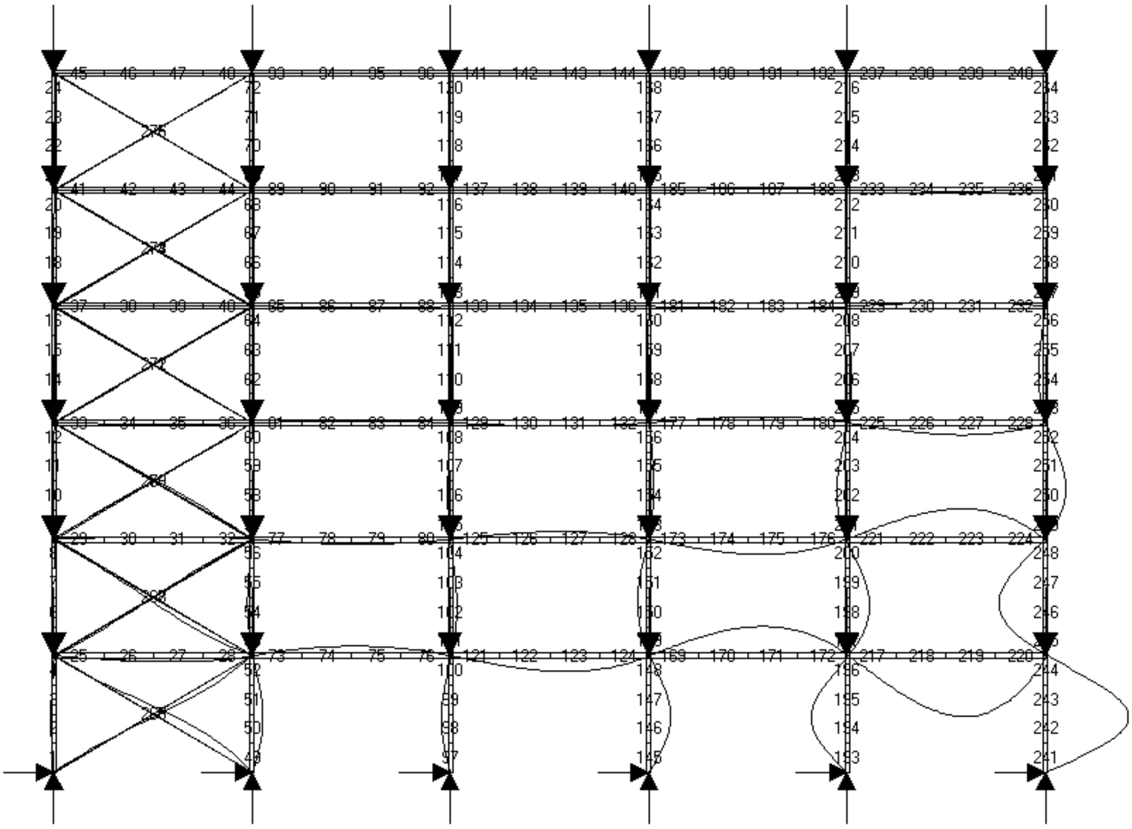


Fig. 1: Fully braced rack, rear-flange uprights, element numbers and critical buckling mode (LBA)

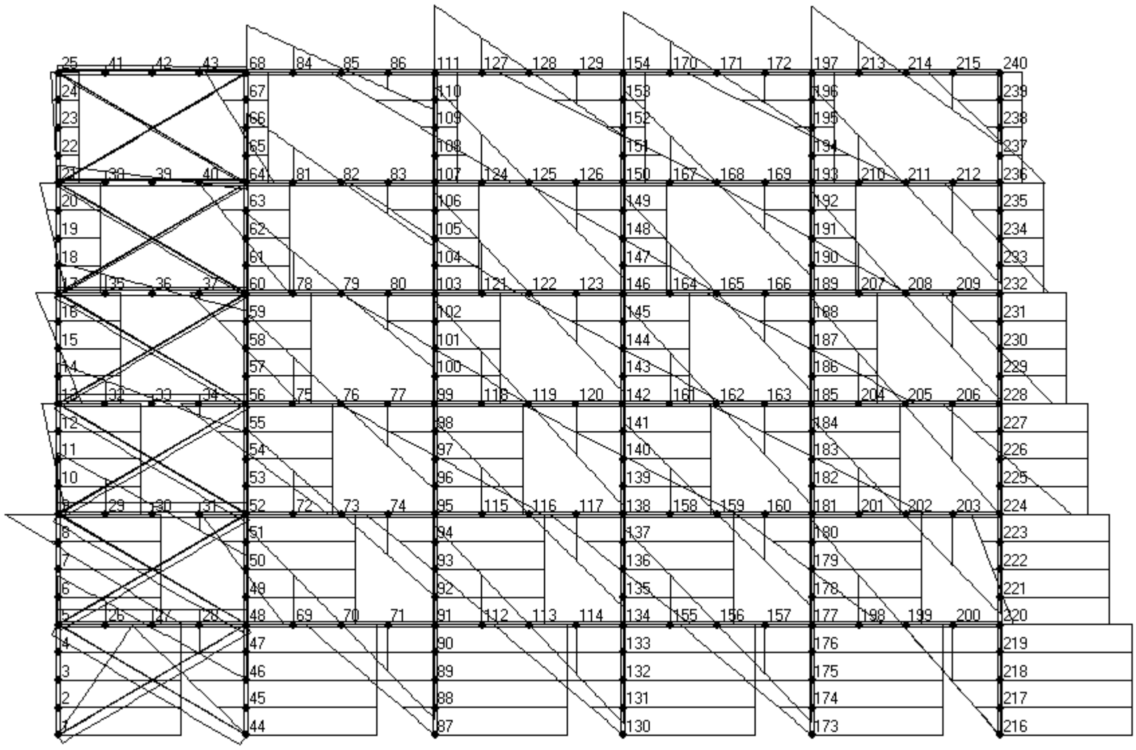


Fig. 2: Node numbers, and axial force and bending moment diagrams (LA)

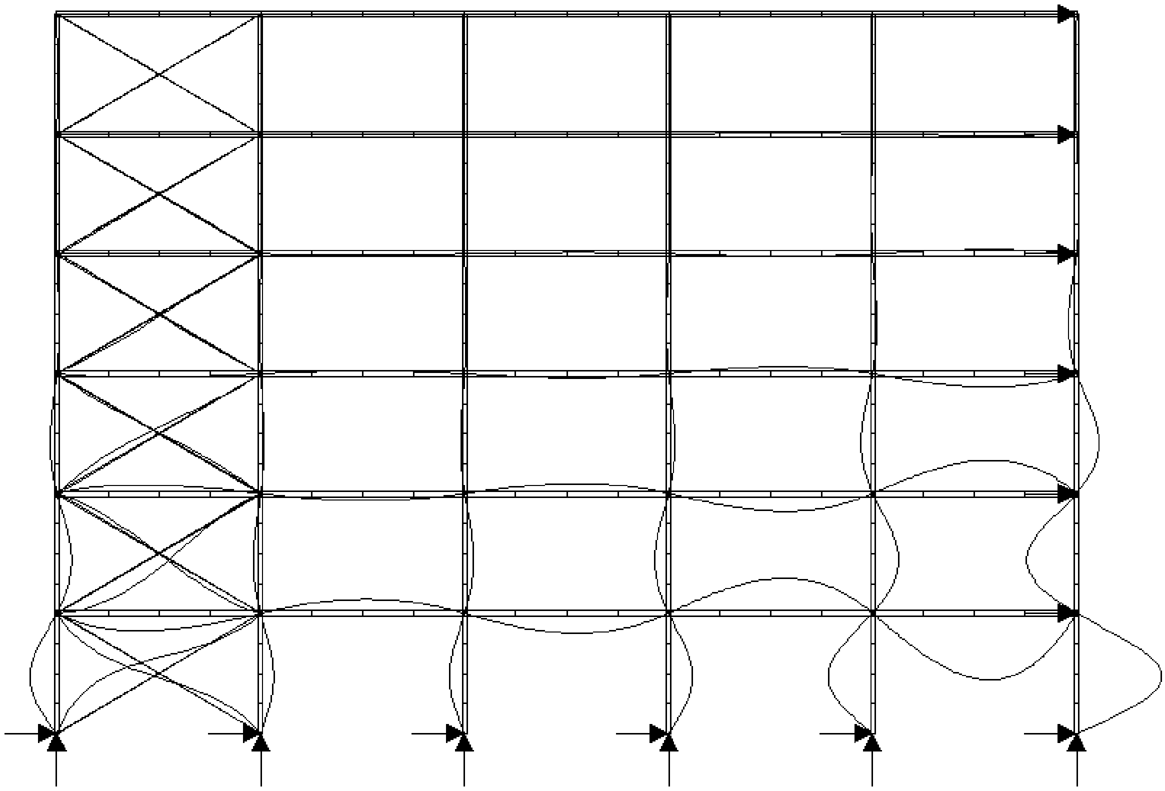


Fig. 3: Buckling mode when all beam levels are restrained horizontally (LBA)

<u>Required:</u> The fully braced steel storage rack shown in Fig. 1 consists of five bays, each 3.4m wide, and six beam levels, equally spaced at 2m vertically. The rack is assumed to be pin-ended at the base and all pallet beam to upright connections are assumed rigid. The

uprights, beams and brace members are Grade 450 RF11015, SHS60x60x4 and CHS30x2 respectively. The rack uprights are subjected to equal forces (P) at all joints between uprights and pallet beams. The horizontal forces representing the effect of out-of-plumb is taken as 0.003V in accordance with the draft Australian standard for Steel Storage Racks, where V is the total vertical force acting at the particular beam level, (V=6P in this example).

The rack is to be designed to the draft Australian standard. The design will be based on LA, GNA and GMNIAs analyses. The objective of this example is to compared the capacities obtained using these three analysis approaches for a fully braced steel storage rack.

Units:

$m := 1L \quad sec := 1T \quad kg := 1M \quad mm := \frac{m}{1000} \quad N := 1M \cdot 1\frac{L}{1T^2} \quad MPa := \frac{N}{mm^2} \quad kN := N \cdot 10^3$

Section properties:

Upright geometry:

Note: A_u, I_{ux} and I_{uy} are the area and 2nd moments of area of the chord. The y-axis is the axis of symmetry.

$A_u := 508.5mm^2$

$I_{ux} := 4.460 \cdot 10^5 \cdot mm^4 \quad r_{ux} := \sqrt{\frac{I_{ux}}{A_u}} \quad r_{ux} = 29.616\,mm \quad y_{max} := 80 \cdot mm - 31.21 \cdot mm$

$y_{max} = 48.79\,mm$

$I_{uy} := 8.484 \cdot 10^5 \cdot mm^4 \quad r_{uy} := \sqrt{\frac{I_{uy}}{A_u}} \quad r_{uy} = 40.847\,mm \quad x_{max} := \frac{110}{2} \cdot mm$

$Z_{ux} := \frac{I_{ux}}{y_{max}} \quad Z_{uy} := \frac{I_{uy}}{x_{max}} \quad x_{max} = 55\,mm$

$Z_{ux} = 9.141 \times 10^3\,mm^3 \quad Z_{uy} = 1.543 \times 10^4\,mm^3$

$J := 381.4 \cdot mm^4 \quad I_w := 1.301 \times 10^9 \cdot mm^6 \quad y_0 := 67.57 \cdot mm$

$r_{o1} := \sqrt{r_{ux}^2 + r_{uy}^2 + y_0^2} \quad r_{o1} = 84.328\,mm$

$\beta_x := -151.7 \cdot mm$

Beam geometry:

$b_b := 60mm \quad t_b := 4mm \quad r_{ob} := 4 \cdot mm$

$A_b := 896mm^2 \quad I_b := 4.707 \cdot 10^5 \cdot mm^4 \quad r_{ib} := r_{ob} - t_b$

$r_b := \sqrt{\frac{I_b}{A_b}} \quad r_b = 22.92\,mm$

Spine bracing geometry:

$d_s := 30mm \quad t_s := 2mm$

$A_s := 175.9\text{mm}^2 \qquad I_s := 1.733605 \cdot 10^4 \cdot \text{mm}^4$

$r_s := \sqrt{\dfrac{I_s}{A_s}} \qquad r_s = 9.928\ \text{mm}$

Material properties of all members, (cold-formed Grade 450 steel):

Upright $f_{yu} := 450\text{MPa}$ Beam $f_{yb} := 450\text{MPa}$ Brace $f_{ys} := 450\text{MPa}$

$E := 210000\text{MPa} \qquad \nu := 0.3 \qquad G := \dfrac{E}{2 \cdot (1 + \nu)} \qquad G = 8.077 \times 10^4\ \text{MPa}$

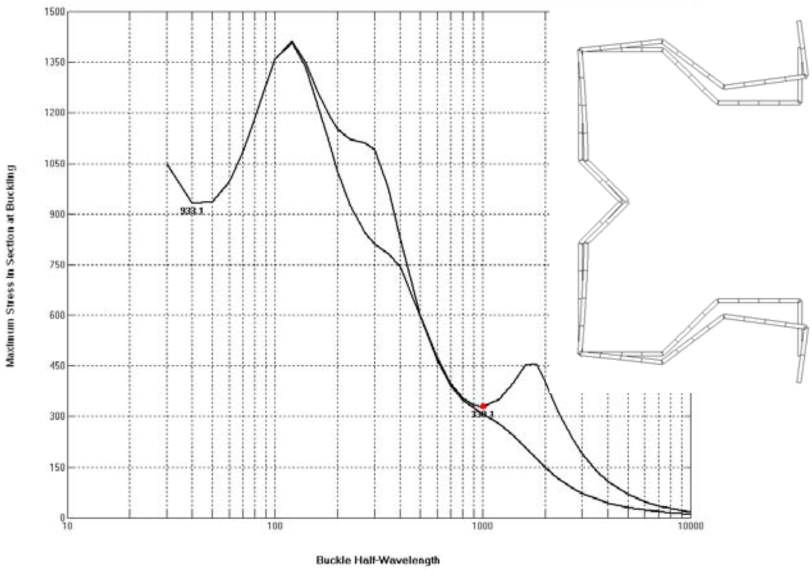


Fig. 4: Buckling stress vs half-wavelength for RF11015 section, axial compression, 1st and 2nd mode of buckling

Thinwall has been used to determined the local and distortional buckling stresses for axial compresion and bending about the x- and y-axes. The buckling stress versus buckle half-wavelength is shown in Fig. 4 for axial compression. The distortional buckling minimum is found (as the second mode of buckling) at a half-wavelength of 1000 mm.
The symmetry axis is the y-axis.

$f_{ol} := 933 \cdot \text{MPa} \qquad f_{od} := 330 \cdot \text{MPa}$

$f_{olx} := 1035 \cdot \text{MPa} \qquad f_{odx} := 450 \cdot \text{MPa}$

$f_{oly} := 946 \cdot \text{MPa} \qquad f_{ody} := 449 \cdot \text{MPa}$

1 Design based on LA analysis

Torsion plays a significant role in the design because the critical column buckling mode is flexural-torsional. The effective lengths for torsion are determined in a manner consistent with the modelled connection at the base of the uprights, which prevents torsion and warping, and the connections between uprights and pallet beams, which prevent torsion and to a small extent warping. Accordingly, the effective length for torsion will be assumed to be 0.7L for the uprights between the floor and the first beam level, and will be assumed to be 0.9L for the uprights between the first and second beam levels. Because of the different effective lengths for torsion, the capacities of the critical uprights in the two lowest levels of the frame need to be determined.

For the uprights between the floor and the first beam level, the maximum axial force and bending moment develop at node 177 in Element 196 of the 2nd right-most upright (here termed Member 1) at the first beam level, as shown in Fig. 2.

For the uprights between the first and second beam levels, the critical member (Member 2) is the second right-most upright (containing Element 200).

The axial force and bending moments in the critical Members 1 and 2, as determined from an LA analysis, are:

Member 1: N=-6.000P M_{11}= 0 M_{12}=-0.0010 P*m (Element 196 in LA, 2nd upright from right)
Member 2: N=-5.000P M_{21}= 0.0005 M_{22}=-0.0006 P*m (Element 200 in LA, 2nd upright from right)

The elastic buckling load of the unbraced frame (P_{cr}),as determined from an LBA analysis, is 99.55kN. The buckling mode is shown in Fig. 1.

The elastic critical buckling load of the rack (P_{crb}), as determined from an LBA analysis with all beam levels prevented against sidesway, is 95.56kN. The corresponding buckling mode is shown in Fig. 3. The axial load at this buckling load is found from $N_{crb}=c_N P_{crb}$ (approximately).

$P_{cr} := 99.55\,kN$

$c_{N1} := 6.000$ $\quad N_{cr1} := c_{N1} \cdot P_{cr}$ $\quad N_{cr1} = 597.3\,kN$

$c_{N2} := 5.000$ $\quad N_{cr2} := c_{N2} \cdot P_{cr}$ $\quad N_{cr2} = 497.75\,kN$

$P_{crb} := 95.56 \cdot kN$ $\quad N_{crb1} := c_{N1} \cdot P_{crb}$ $\quad N_{crb1} = 573.36\,kN$

$N_{crb2} := c_{N2} \cdot P_{crb}$ $\quad N_{crb2} = 477.8\,kN$

Axial capacity of upright Member 1

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey1} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb1}}} \qquad L_{ey1} = 1.751\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large warping restraint, be taken as 0.7 times the distance between the bracing points. Note that in the FE analysis, the uprights are prevented to warp at the base and restrained against torsion at the base and at the panel points. The warpint restraint is small at the panel points between uprights and pallet beams. Thus,

$$L_{ez1} := 0.7 \cdot 2 \cdot m \qquad L_{ez1} = 1.4\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy1} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey1}}{r_{uy}}\right)^2} \qquad f_{oy1} = 1.128 \times 10^3 \, MPa$$

$$f_{oz1} := \frac{G \cdot J}{A_u \cdot r_{o1}^2} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez1}^2}\right) \qquad f_{oz1} = 388.975 \, MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz1} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy1} + f_{oz1} - \sqrt{\left(f_{oy1} + f_{oz1}\right)^2 - 4 \cdot \beta \cdot f_{oy1} \cdot f_{oz1}}\right] \qquad f_{oyz1} = 312.215 \, MPa$$

$$f_{oc1} := f_{oyz1} \qquad f_{oc1} = 312.215 \, MPa$$

$$\lambda_{c1} := \sqrt{\frac{f_{yu}}{f_{oc1}}} \qquad \lambda_{c1} = 1.201$$

$$f_{n1} := \text{if}\left(\lambda_{c1} < 1.5, 0.658^{\lambda_{c1}^2} \cdot f_{yu}, \frac{0.977}{\lambda_{c1}^2} \cdot f_{yu}\right) \qquad f_{n1} = 246.161 \, MPa$$

Determine columns capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce1} := A_u \cdot f_{n1} \qquad N_{ce1} = 125.173 \, kN$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43 \, kN$$

$$\lambda_l := \sqrt{\frac{N_{ce1}}{N_{ol}}} \qquad \lambda_l = 0.514$$

$$N_{cl} := \text{if}\left[\lambda_l < 0.776, N_{ce1}, \left[1 - 0.15 \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4}\right] \cdot \left(\frac{N_{ol}}{N_{ce1}}\right)^{0.4} \cdot N_{ce1}\right] \qquad N_{cl} = 125.173 \, kN$$

Distortional buckling:

$$N_{yu} := A_u \cdot f_{yu} \qquad N_{yu} = 228.825 \, kN$$

$$N_{od} := A_u \cdot f_{od} \qquad N_{od} = 167.805 \, kN$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \text{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25 \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right] \cdot \left(\frac{N_{od}}{N_{yu}}\right)^{0.6} \cdot N_{yu}\right] \qquad N_{cd} = 150.542 \, kN$$

Column capacity:

$$N_{c1} := \min(N_{ce1}, N_{cl}, N_{cd}) \qquad N_{c1} = 125.173\ kN$$

Axial capacity of upright Member 2

As per Clause 4.2.2.1 of the draft Standard, the effective length for flexural buckling may be back-calculated from the critical bukling load of the corresponding fully braced rack, i.e.based on N_{crb}

$$L_{ey2} := \pi \cdot \sqrt{\frac{E \cdot I_{uy}}{N_{crb2}}} \qquad L_{ey2} = 1.918\ m$$

As per Clause 4.2.2.3 of the draft Standard, the effective length for torsional buckling may, for connections providing large small restraint, be taken as 1.0 times the distance between the bracing points. Note that in the FE analysis, the uprights are restrained against torsion at the panel points, and there is a small degree of warping restraint since warping of the web (only) is restrained. Accordingly, the effective length for torsion will be taken as,

$$L_{ez2} := 0.9 \cdot 2 \cdot m \qquad L_{ez2} = 1.8\ m$$

Determine the column strength according to AS/NZS4600

$$f_{oy2} := \frac{\pi^2 \cdot E}{\left(\frac{L_{ey2}}{r_{uy}}\right)^2} \qquad f_{oy2} = 939.626\ MPa$$

$$f_{oz2} := \frac{G \cdot J}{A_u \cdot r_{o1}^{\,2}} \cdot \left(1 + \frac{\pi^2 \cdot E \cdot I_w}{G \cdot J \cdot L_{ez2}^{\,2}}\right) \qquad f_{oz2} = 238.671\ MPa$$

$$\beta := 1 - \left(\frac{y_0}{r_{o1}}\right)^2$$

$$f_{oyz2} := \frac{1}{2 \cdot \beta} \cdot \left[f_{oy2} + f_{oz2} - \sqrt{(f_{oy2} + f_{oz2})^2 - 4 \cdot \beta \cdot f_{oy2} \cdot f_{oz2}}\right] \qquad f_{oyz2} = 202.824\ MPa$$

$$f_{oc2} := f_{oyz2} \qquad f_{oc2} = 202.824\ MPa$$

$$\lambda_{c2} := \sqrt{\frac{f_{yu}}{f_{oc2}}} \qquad \lambda_{c2} = 1.49$$

$$f_{n2} := \text{if}\left(\lambda_{c2} < 1.5, 0.658^{\lambda_{c2}^{\,2}} \cdot f_{yu}, \frac{0.977}{\lambda_{c2}^{\,2}} \cdot f_{yu}\right) \qquad f_{n2} = 177.794\ MPa$$

Determine column capacity using Section 7 of AS/NZS4600 (Direct Strength Method):

Overall buckling:

$$N_{ce2} := A_u \cdot f_{n2} \qquad N_{ce2} = 90.408\ kN$$

Local buckling:

$$N_{ol} := A_u \cdot f_{ol} \qquad N_{ol} = 474.43\ kN$$

$$\lambda_l := \sqrt{\frac{N_{ce2}}{N_{ol}}} \qquad \lambda_l = 0.437$$

$$N_{cl} := \text{if}\left[\lambda_l < 0.776, N_{ce2}, \left[1 - 0.15\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\right]\cdot\left(\frac{N_{ol}}{N_{ce2}}\right)^{0.4}\cdot N_{ce2}\right] \qquad N_{cl} = 90.408\,\text{kN}$$

Distortional buckling:

$$N_{yu} := A_u \cdot f_{yu} \qquad N_{yu} = 228.825\,\text{kN}$$

$$N_{od} := A_u \cdot f_{od} \qquad N_{od} = 167.805\,\text{kN}$$

$$\lambda_d := \sqrt{\frac{N_{yu}}{N_{od}}} \qquad \lambda_d = 1.168$$

$$N_{cd} := \text{if}\left[\lambda_d < 0.561, N_{yu}, \left[1 - 0.25\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\right]\cdot\left(\frac{N_{od}}{N_{yu}}\right)^{0.6}\cdot N_{yu}\right] \qquad N_{cd} = 150.542\,\text{kN}$$

Column capacity:

$$N_{c2} := \min(N_{ce2}, N_{cl}, N_{cd}) \qquad N_{c2} = 90.408\,\text{kN}$$

Flexural capacities of upright Members 1 and 2

The upright members are bent about the symmetry y-axis. As such, they are ordinarily subject to flexural-torsional buckling, involving flexure about the x-axis and torsion. However, in this example, the uprights are assumed to be braced in the cross-aisle x-direction. The flexural capacity for bending about the y-axis is thus the yield moment.

Since the cross-section is slender, local and distortional buckling need to be accounted for. This is achieved using the Direct Strength Method.

Section capacity:

$$M_{suy} := f_{yu} \cdot Z_{uy} \qquad M_{suy} = 6.941\,\text{kN}\cdot\text{m}$$

Overall buckling:

$$M_{bey} := M_{suy} \qquad M_{bey} = 6.941\,\text{kN}\cdot\text{m}$$

Local buckling:

$$M_{oly} := Z_{uy} \cdot f_{oly} \qquad M_{oly} = 14.592\,\text{m}\,\text{kN}$$

$$\lambda_{ly} := \sqrt{\frac{M_{bey}}{M_{oly}}} \qquad \lambda_{ly} = 0.69$$

$$M_{bly} := \text{if}\left[\lambda_{ly} < 0.776, M_{bey}, \left[1 - 0.15\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\right]\cdot\left(\frac{M_{oly}}{M_{bey}}\right)^{0.4}\cdot M_{bey}\right] \qquad M_{bly} = 6.941\,\text{kN}\cdot\text{m}$$

Distortional buckling:

$$M_{yuy} := Z_{uy} \cdot f_{yu} \qquad M_{yuy} = 6.941\,\text{kN}\cdot\text{m}$$

$$M_{ody} := Z_{uy} \cdot f_{ody} \qquad M_{ody} = 6.926\,kN \cdot m$$

$$\lambda_{dy} := \sqrt{\frac{M_{yuy}}{M_{ody}}} \qquad \lambda_{dy} = 1.001$$

$$M_{bdy} := if\left[\lambda_{dy} < 0.673, M_{yuy}, \left[1 - 0.22 \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5}\right] \cdot \left(\frac{M_{ody}}{M_{yuy}}\right)^{0.5} \cdot M_{yuy}\right] \qquad M_{bdy} = 5.41\,m\,kN$$

Bending capacity (y-axis bending):

$$M_{by} := min\left(M_{bey}, M_{bly}, M_{bdy}\right) \qquad M_{by} = 5.41\,kN \cdot m$$

Combined compression and flexural capacity of upright members.

AS/NSZS4600 specifies a linear interaction equation for determining the member strength under the combined actions of compresion and bending, as follows:

$N^*/(\phi_c\ N_c) + C_{my} M_y^*/(\phi_b\ M_{by} \alpha_y) < 1$

where M_y^* is the maximum bending moment in the member considered, as determined from an LA analysis. In this equation, moment amplification is accounted for through the terms C_m and α, where,

$\alpha = 1-N^*/N_e \quad => \quad 1/\alpha = N_e/(N_e-N^*)$

In this equation, N_e is the flexural buckling load, as determined from an LBA analysis. It is seen that the factor $1/\alpha$ is, in fact, the same amplification factor as that used in Clause 3.3.9 of the draft standard for steel storage racks.

AS/NZS4600 allows a value of C_m of 0.85 to be used for sway frames. However, to be consistent with Clause 3.3.9 of the draft standard, C_m is (conservatively) taken as unity so that the amplification factor becomes $1/\alpha = N_e/(N_e-N^*)$.

Member 1:

We have $N^*=c_{N1}*P$, $c_{N1}=6.000$, $M_{11y}^*=0$ and $M_{12y}^*=c_{My}*P*m$, $c_{My1}=-0.0010$; and $\alpha_n=1-N^*/N_e$. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB. Note that for unbraced frames, AS/NZS4600 specifies $C_m=0.85$.

$$c_{My1} := 0.0010 \cdot m$$

$$C_m := 1.0$$

$$\phi_c := 0.85 \qquad \phi_b := 0.9$$

$$AA_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1} \cdot P_{cr}} \quad BB_1 := \frac{c_{N1}}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}} \qquad BB_1 = 0.067\ \frac{kg}{A^2 \cdot m^3 \cdot s^4}$$

$$P_1 := \frac{1}{2 \cdot AA_1} \cdot \left(BB_1 - \sqrt{BB_1^{\,2} - 4 \cdot AA_1}\right) \qquad P_1 = 17.655\,kN$$

$$check := \frac{c_{N1} \cdot P_1}{\phi_c \cdot N_{c1}} + \frac{c_{My1} \cdot P_1 \cdot C_m}{\phi_b \cdot M_{by} \cdot \left(1 - \frac{P_1}{P_{cr}}\right)} \qquad check = 1$$

Member 2:

We have N*=c_{N2}*P, c_{N2}=5.000, M_{21y}*=0.0005*P*m and M_{22y}*=c_{My}*P*m, c_{My2}=-0.0006; and α_n=1-N*/N_e. The interaction equation leads to a quadratic in P which has been solved using auxiliary parameters AA and BB.

$$c_{My2} := 0.0006 \cdot m$$

$$AA_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2} \cdot P_{cr}} \quad BB_2 := \frac{c_{N2}}{\phi_c \cdot N_{c2}} + \frac{c_{My2} \cdot C_m}{\phi_b \cdot M_{by}} + \frac{1}{P_{cr}}$$

$$P_2 := \frac{1}{2 \cdot AA_2} \cdot \left(BB_2 - \sqrt{BB_2^{\,2} - 4 \cdot AA_2}\right) \qquad P_2 = 15.335\,kN$$

Design capacity of storage rack based on LA analysis:

Considering the capacities of members 1 and 2, the maximum factored design load (P) is the minimum of the determined values of P:

$$P_1 = 17.655\,kN \qquad P_2 = 15.335\,kN$$

$$P_{min} := \min\left(P_1, P_2\right) \qquad P_{min} = 15.335\,kN \qquad P_{LA} := P_{min}$$

2 Design based on GNA analysis

The maximum design actions develop near the base of the right-most upright. In the GNA analysis, the axial force (N) and bending moment (M) are nonlinear functions of the applied force (P).

The axial member capacity (N_c) and bending capacity (M_{by}) are determined according to AS/NSZS4600 using the same procedure as that detailed under LA analysis. However, the interaction equation changes since the bending moment does not need amplification when determined from a GNA analysis. It takes the linear form:

$$N^*/(\phi_c\, N_c) + M^*/(\phi_b\, M_b) < 1$$

where M* is the maximum bending moment in the member considered.

The (N*,M*) values computed from the GNA analysis are tabulated below for increasing values of loading (P). For each set of values, the left-hand side of the interaction equation is computed. When this exceeds unity, the capacity of the rack is exhausted. The corresponding value of P is the factored capacity of the rack.

Data := GNA - braced - PRFSA.xls

$$\text{Data} = \begin{pmatrix} 10 & 9.4\times10^{-3} & 60 & 5.8\times10^{-3} & 50 \\ 12.5 & 0.012 & 75 & 7.3\times10^{-3} & 62.5 \\ 15 & 0.014 & 90 & 8.9\times10^{-3} & 75 \\ 20 & 0.018 & 120 & 0.012 & 100 \end{pmatrix}$$

$P :=$ for $i \in 0..3$

$\quad ss_i \leftarrow \text{Data}_{i,0}\cdot kN$

$\quad ss$

Element 196 (2nd right-most upright, between floor and 1st beam level):

$LHS1 :=$ for $i \in 0..3$

$\quad N \leftarrow \text{Data}_{i,2}\cdot kN$

$\quad M \leftarrow \text{Data}_{i,1}\cdot kN\cdot m$

$\quad ss_i \leftarrow \dfrac{N}{\phi_c\cdot N_{c1}} + \dfrac{M}{\phi_b\cdot M_{by}}$

$\quad ss$

$$P = \begin{pmatrix} 10 \\ 12.5 \\ 15 \\ 20 \end{pmatrix} kN \qquad LHS1 = \begin{pmatrix} 0.566 \\ 0.707 \\ 0.849 \\ 1.131 \end{pmatrix}$$

Element 200 (2nd right-most upright, between 1st and 2nd beam level)::

$LHS2 :=$ for $i \in 0..3$

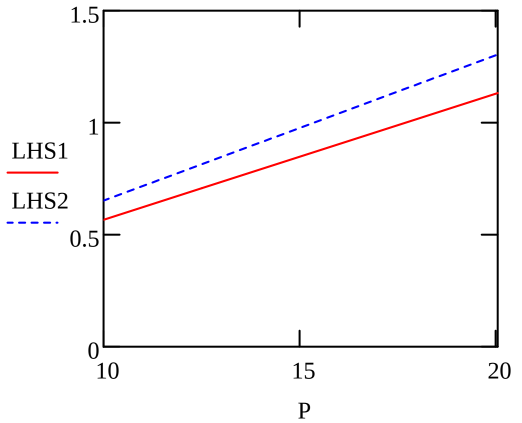
$\quad N \leftarrow \text{Data}_{i,4}\cdot kN$

$\quad M \leftarrow \text{Data}_{i,3}\cdot kN\cdot m$

$\quad ss_i \leftarrow \dfrac{N}{\phi_c\cdot N_{c2}} + \dfrac{M}{\phi_b\cdot M_{by}}$

$\quad ss$

$$P = \begin{pmatrix} 10 \\ 12.5 \\ 15 \\ 20 \end{pmatrix} kN \qquad LHS2 = \begin{pmatrix} 0.652 \\ 0.815 \\ 0.978 \\ 1.304 \end{pmatrix}$$



Determine the value of P producing a LHS of unity by interpolation:

$n_u := 2 \qquad x_1 := P_{n_u} \qquad x_2 := P_{n_u+1} \qquad y_1 := LHS2_{n_u} \qquad y_2 := LHS2_{n_u+1}$

$P_u := \dfrac{1 - y_1}{y_2 - y_1}\cdot(x_2 - x_1) + x_1 \qquad x_1 = 15\,kN \qquad y_1 = 0.978$

$P_u = 15.341\,kN \qquad P_{GNA} := P_u$

3 Design based on GMNIAs analysis

The ultimate load (P) obtained directly from a GMNIAs analysis is:

$$P_{max} := 20.0 \cdot kN$$

Assuming a resistance factor for the rack of ϕ=0.9, the factored ultimate load is obtained as:

$$\phi := 0.9$$

$$P_{GMNIAs} := \phi \cdot P_{max} \qquad P_{GMNIAs} = 18\,kN$$

4 Summary

The factored ultimate loads (P) obtained on the basis of LA, GNA and GMNIAs analyses are:

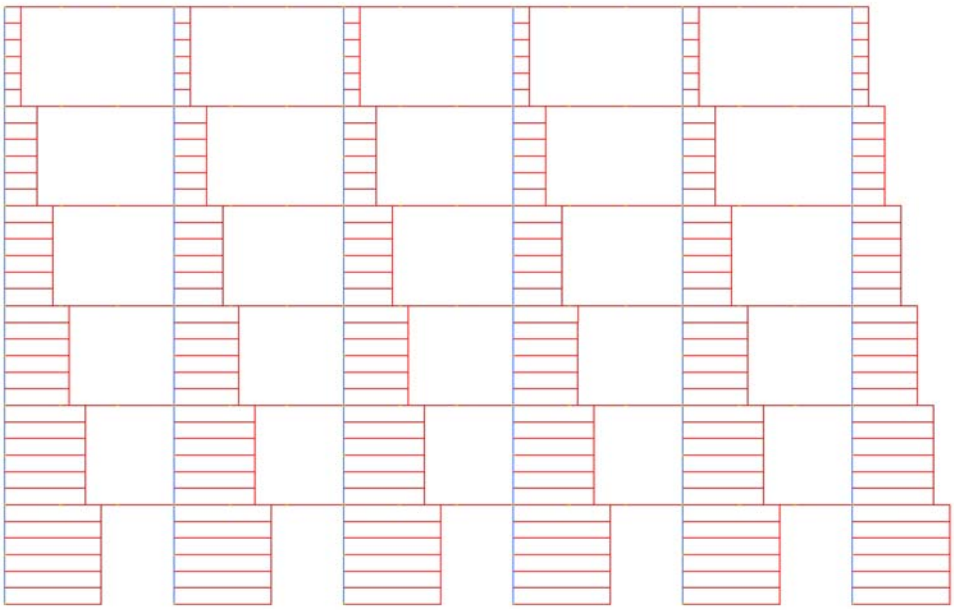
$$P_{LA} = 15.335\,kN \qquad P_{GNA} = 15.341\,kN \qquad P_{GMNIAs} = 18\,kN$$

The factored ultimate load (18kN) determined on the basis of a GMNIAs analysis is 14.8% and 14.7% higher than those (15.335kN and 15.341kN) obtained using LA and GNA analyses, respectively.

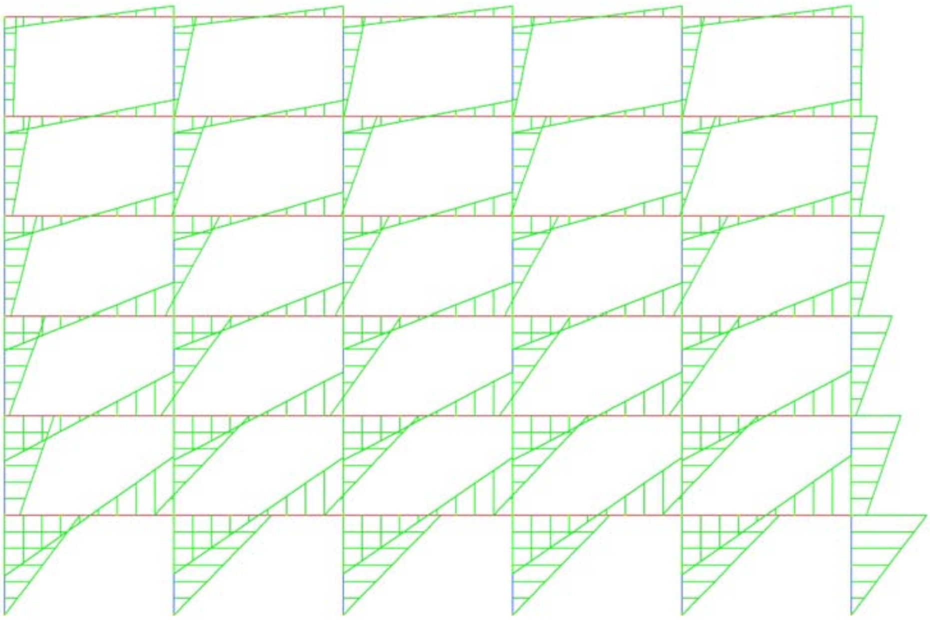
APPENDIX 2

Linear Analysis axial force and bending moment distributions

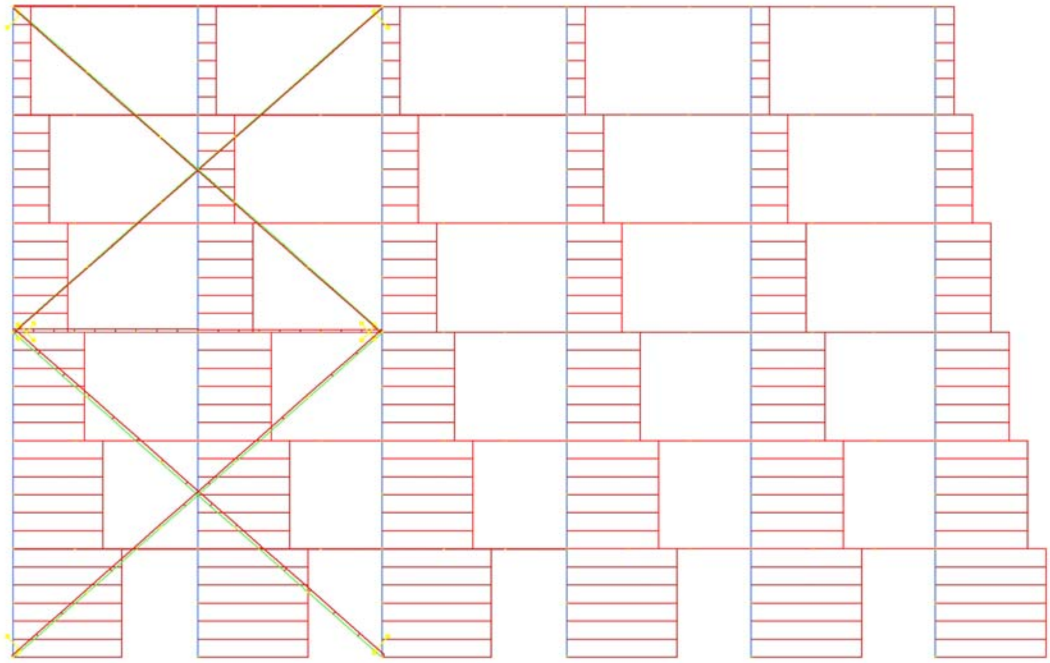
100x100x6 mm SHS upright – Unbraced rack – Axial force distribution



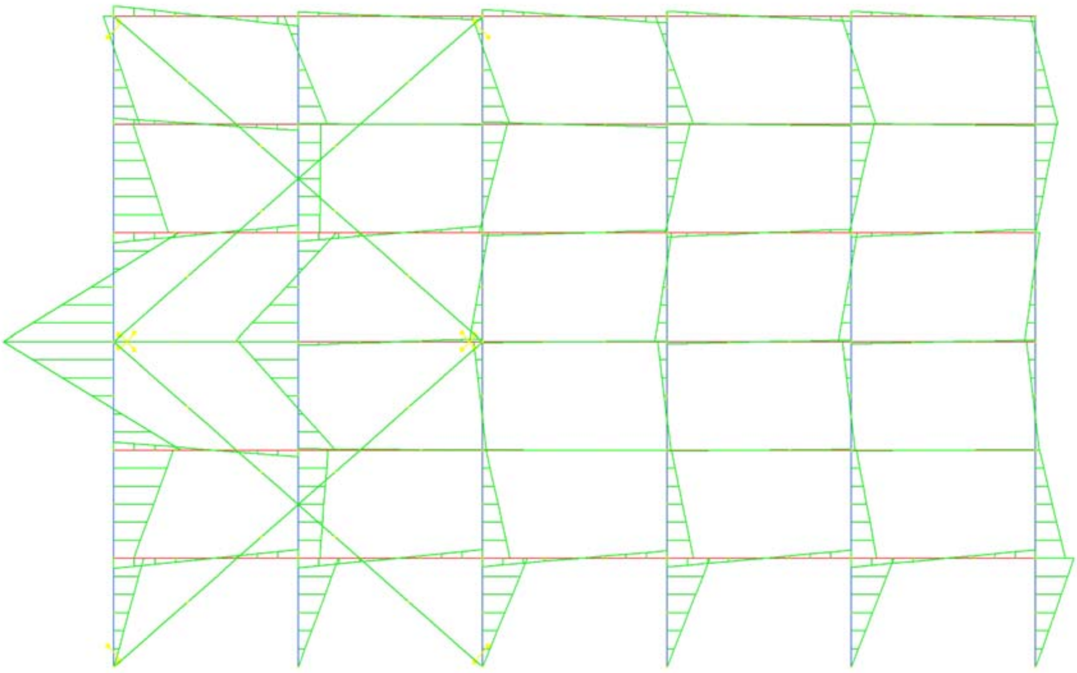
100x100x6 mm SHS upright – Unbraced rack – Bending moment distribution



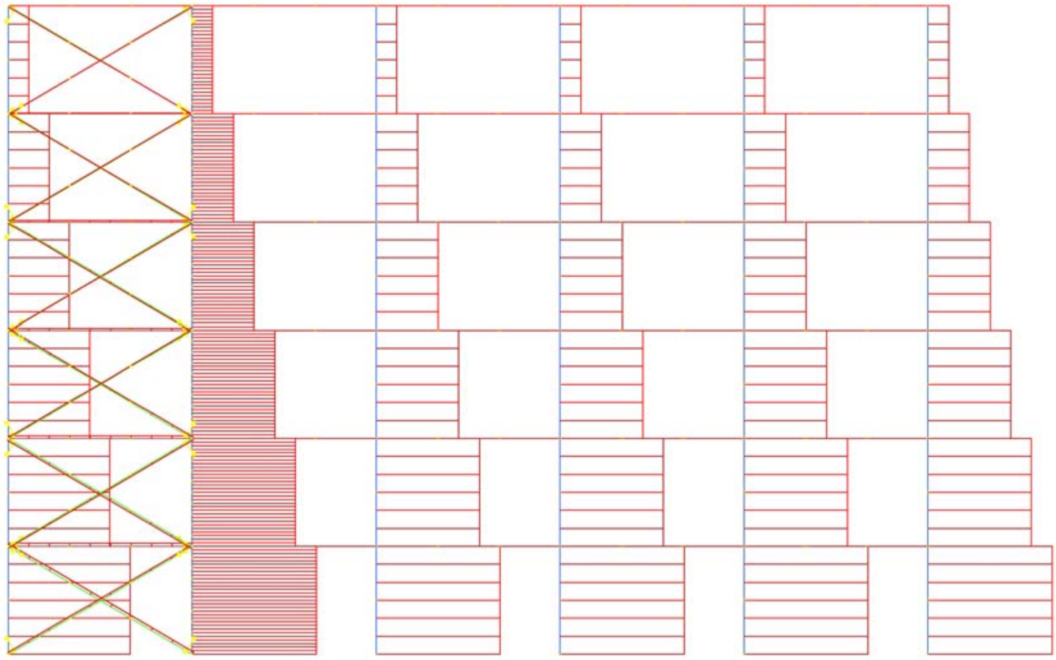
100x100x6 mm SHS upright – Semi-braced rack – Axial force distribution



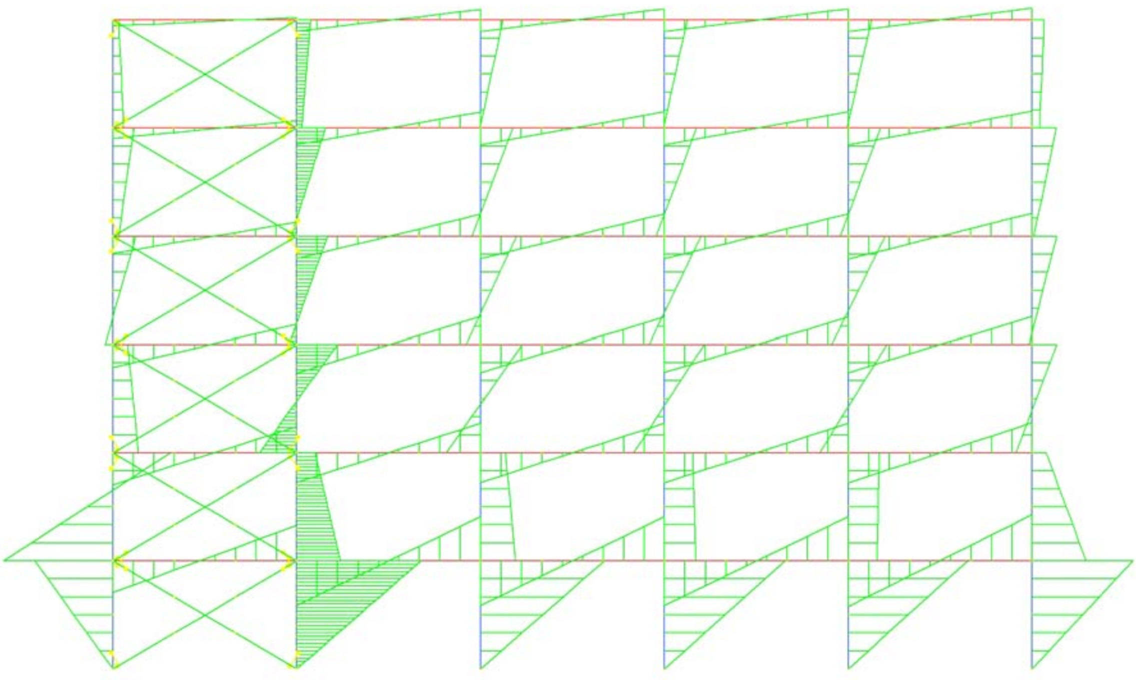
100x100x6 mm SHS upright – Semi-braced rack – Bending moment distribution



100x100x6 mm SHS upright – Fully-braced rack – Axial force distribution



100x100x6 mm SHS upright – Fully-braced rack – Bending moment distribution



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