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Research Report No R890

N S Trahair BSc BE MEngSc PhD DEng

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Abstract:

Single equal angle steel beams are often used as lintels to support eccentric loading acting normal to one flange. This loading causes combined biaxial bending and torsion, which is not allowed for in most steel design codes. Instead, approximate methods based somewhat loosely on past research studies have been used to develop design approximations and tables.

This paper reviews past research on single equal angle beams used as lintels and develops an improved method of predicting their strengths which includes the effects of initial twist rotations, eccentric loads, and large twist rotations, and utilizes the plastic capacities of compact beams. The strengths predicted are significantly higher than those of previous approximations. More accurate strength approximations are proposed, and suggestions are made for serviceability design.

Keywords: angles, beams, bending, buckling, design, elasticity, member strength, moments, serviceability, steel, torsion.

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1 INTRODUCTION

Single equal angle steel beams are often used as lintels to support eccentric loading normal to one flange, as shown in Fig. 1. This loading does not act parallel to a principal plane and so causes combined biaxial bending and torsion, which is not allowed for in most steel design codes (AISC, 2005a; BSI, 2000, 2005; SA, 1998). Instead, approximate methods based somewhat loosely on past research studies (Leigh and Lay, 1969, 1970a, b; Goh, Dayawansa, and Bennetts, 1991) have been used to develop design approximations (AISC, 2005a) and tables (Australian Institute of Steel Construction, 1987, 1999).

The behaviour of lintels depends on their loading and restraint. Lintels which are restrained laterally and prevented from twisting may fail by yielding or local buckling, and can easily be designed for the primary bending moments or for shear and bearing forces (Trahair, 2002a, b). Unrestrained lintels are not so easily designed, because non-linearities, such as those which cause lateral buckling and those due to monosymmetry, enhance the primary twist rotations, which themselves enhance the bending moments. These enhancements are difficult to predict.

This paper reviews past research on unrestrained compact single equal angle beams used as lintels. It develops an improved method of predicting their enhanced bending moments which includes the effects of eccentric loads, lateral buckling influences, monosymmetry, initial twist rotations, and large twist rotations. The predictions of this method are used in formulations of the plastic capacities of compact beams to predict their nominal strengths. These are significantly higher than those of the AISC (2005a). Simple approximations for the nominal design strengths are proposed. Serviceability design is briefly considered.

2 PREVIOUS RESEARCH

2.1 Leigh and Lay (1969, 1970a)

In their 1970a report, Leigh and Lay extended their previous research (1969) on the biaxial bending of straight steel equal angle beams loaded in uniform bending in the plane of one of the angle legs to unequal angles. They developed exact equations for the small deformation elastic non-linear deflections and twist rotations. They used these to predict the moments developed, even though some very large twist rotations were calculated.

Leigh and Lay (1970a) used their moment calculations to develop working stress design recommendations based on “first yield” at a maximum permissible stress of $0.66 f_y$ (in which f_y is the yield stress), which ignores significant reserves of strength at full plasticity but does not fully allow for the increased non-linearities that occur near failure. The effects of geometrical imperfections such as initial crookedness or twist were not considered in the non-linear analysis, although initial twist rotations and linear twist rotations caused by eccentric loads were considered for addition to the non-linear twist rotations.

This research would be improved by including the additional twist rotations (initial and those due to eccentric loads) in the non-linear analysis, by replacing the working stress “first yield” strength criterion by one of full plastic failure of the most heavily loaded cross-section, and by allowing for the large twist rotations that develop.

2.2 Goh, Dayawansa, and Bennetts (1991)

Goh, Dayawansa, and Bennetts sought to revise the working stress design findings of Leigh and Lay (1969, 1970a, b) for limit states design. For this they carried out iterative numerical non-linear analyses of the elastic biaxial bending and torsion of simply supported single angle beams with uniformly distributed loads acting near the centre of the horizontal flange at an eccentricity of $b/2$ from the shear centre. Initial crookedness and twist were ignored. Their method includes the effects of large rotations and load height but appears to ignore any monosymmetry of the cross-section.

They considered that the beam capacity was reached when the most highly loaded section of the beam just satisfied an approximate biaxial bending section capacity equation which for compact beams was based on a combination of the principal axis full plastic moments. They found that equal angles were significantly stronger when the horizontal leg was down than when it was up.

This research would be improved by including monosymmetry effects and initial twist rotations in the non-linear analysis, and by replacing the biaxial bending section capacity criterion by one of full plastic failure of the most heavily loaded cross-section.

2.3 Trahair (2007)

Trahair (2007) developed an approximation for the elastic non-uniform biaxial bending and torsion of equal angle beams under uniform bending and linear torque, from which he developed an approximation for the strengths of single angle beams under uniformly distributed eccentric loads. The effects on initial twisting and load height were included but the monosymmetry of the section was ignored. The initial twist magnitudes were such that when the method was applied to beams loaded in the stiffer principal plane it would predict the lateral buckling design strengths proposed in Trahair (2003) which allowed for the effects of geometrical and material imperfections. The effects of the moment distribution and load height on the lateral buckling strengths were allowed for by making appropriate adjustments to the elastic lateral buckling moments.

He considered that the capacity of a compact beam was reached when the most highly loaded section of the beam just satisfied the exact fully plastic biaxial bending section capacity equation (Trahair, 2002a). He also found that equal angles were significantly stronger when the horizontal leg was down than when it was up.

This research would be improved by including monosymmetry effects and allowing for the large twist rotations that develop.

3 ELASTIC ANALYSIS

3.1 General

An elastic simply supported equal angle section beam of length L and initial twist ϕ_i given by

$$\phi_i = \phi_{i0} (1 - 4z^2 / L^2) \quad (1)$$

in which z is the distance along the beam measured from mid-span, is shown in Fig. 2. The beam has equal and opposite end moments M , M causing uniform bending in the initial yz , xz principal planes (so that the resultant moment $\sqrt{2}M$ acts in the plane of the horizontal leg) and a uniformly distributed torque per unit length m . These moments provide a conservative model for the more common uniformly distributed loading shown in Fig. 1.

The large twist rotation differential equations of equilibrium for biaxial bending and torsion are

$$\begin{Bmatrix} -EI_x v^{ii} \\ EI_y u^{ii} \\ GJ(\phi - \phi_i)^i \end{Bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ u^i & v^i + \beta_y \phi^i & 1 \end{bmatrix} \begin{Bmatrix} M \\ M \\ M_z \end{Bmatrix} \quad (2)$$

in which i indicates differentiation with respect to z ,

$$M_z = -m z \quad (3)$$

is the variation of the axial torque caused by the distributed torque m , E and G are the Young's and shear moduli of elasticity, I_x and I_y are the second moments of area about the principal x , y axes, u and v are the shear centre deflections in the x , y directions, ϕ is the total angle of twist rotation, and β_y is the monosymmetry section constant given by

$$\beta_y = \frac{1}{I_y} \int_A x(x^2 + y^2) dA - 2x_0 \quad (4)$$

in which x_0 is the shear centre coordinate.

In Equations 2, the left hand sides represent the internal resistances to bending and torsion, while the right hand sides represent the first- and second-order actions resulting from the applied actions M , M , and M_z , the small deflections u , v , and the twist rotations ϕ . The first two of Equations 2 omit second-order moment components of the applied torque M_z which Trahair and Teh (2001) found to be small. The third of Equations 2 omits a large twist rotation resistance $EI_n(\phi^i)^3/2$ (Trahair, 2005) because the non-linear "Wagner" section constant $I_n = b^5 t / 90$ is quite small for equal angle sections, in which b is the leg length and t the thickness of the section.

The deflections u , v can be eliminated from Equations 2, whence

$$\begin{aligned}\phi^{ii} + \sin \phi \frac{M^2 (1/EI_y + 1/EI_x)}{(GJ - M\beta_y)} \\ = \cos \phi \frac{M^2 (1/EI_y - 1/EI_x)}{(GJ - M\beta_y)} - \frac{8GJ(\phi_{i0} + \phi_{e0})/L^2}{(GJ - M\beta_y)}\end{aligned}\quad (5)$$

in which

$$\phi_{e0} = mL^2 / 8GJ \quad (6)$$

The principal axis bending moments are greatest at mid-span, and can be obtained from Equations 2 as

$$M_x = M (\cos \phi_0 + \sin \phi_0) \quad (7a)$$

$$M_y = M (\cos \phi_0 - \sin \phi_0) \quad (7b)$$

in which ϕ_0 is the value of ϕ at mid-span.

3.3 Small Twist Rotations

If the twist rotations are small, then the terms $\cos \phi$ and $\sin \phi$ can be replaced by 1 and ϕ respectively, and when there is no initial twist ($\phi_i = 0$) and no continuous torque ($\phi_{e0} = 0$), then Equation 5 becomes

$$\phi^{ii} + \phi \frac{M^2 (1/EI_y + 1/EI_x)}{(GJ - M\beta_y)} = \frac{M^2 (1/EI_y - 1/EI_x)}{(GJ - M\beta_y)} \quad (8)$$

The solution of Equation 8 which satisfies the boundary conditions $(\phi^i)_0 = (\phi)_{L/2} = 0$ for simple supports is given by

$$\phi = -\frac{a_1^2}{a_2^2} \left(\frac{\cos a_2 z - \cos a_2 L/2}{\cos a_2 L/2} \right) \quad (9)$$

in which

$$a_1^2 = \frac{M^2 (1/EI_y - 1/EI_x)}{(GJ - M\beta_y)} \quad (10a)$$

$$a_2^2 = \frac{M^2 (1/EI_y + 1/EI_x)}{(GJ - M\beta_y)} \quad (10b)$$

The central total twist rotation at $z = 0$ is

$$\phi_0 = -\frac{a_1^2}{a_2^2} \left(\frac{1 - \cos a_2 L / 2}{\cos a_2 L / 2} \right) \quad (11)$$

which is always negative (counter-clockwise in Fig. 2b) and approaches infinity at a limiting moment given by

$$M_L = M_{yz} \left\{ \sqrt{0.8 + \left(\frac{0.4M_{yz}\beta_y}{GJ} \right)^2} - \frac{0.4M_{yz}\beta_y}{GJ} \right\} \quad (12)$$

in which

$$M_{yz} = \sqrt{\pi^2 EI_y GJ / L^2} \quad (13)$$

Equations 9–13 are equivalent to those obtained by Leigh and Lay (1969, 1970a).

The small rotation central principal axis moments are

$$M_x = M(1 + \phi_0) \quad (14a)$$

$$M_y = M(1 - \phi_0) \quad (14b)$$

The small rotation behaviour has been evaluated for the example 95 x 95 x 10 x 8000 equal angle beam whose properties are given in Fig. 3. The beam's horizontal flange is down, and the monosymmetry section constant β_y is positive. In this case the effect of monosymmetry is to decrease the effective torsional rigidity from GJ to $GJ - M\beta_y$, and consequently to increase the (negative) twist rotations and decrease the limiting moment M_L in comparison with those for a beam whose horizontal flange is up (negative β_y).

The variations of the central twist rotation $-\phi_0$ with the values of the applied moments M are shown in Fig. 4. It can be seen that the (negative) twist rotation commences at the beginning of loading, and that while it is very small at first, it increases rapidly and becomes very large near the limiting moment $M_L = 8.34$ kNm. These large rotations violate the assumption of small rotations and the approximations of $\cos\phi = 1$ and $\sin\phi = \phi$ are no longer valid.

Also shown in Fig. 4 are the variations of the central principal axis moments M_x , M_y with the value of the central twist rotation $-\phi_0$. At low values of $-\phi_0$, these are nearly equal to the applied moments M but they diverge as $-\phi_0$ increases, with M_y accelerating, but M_x decelerating and then decreasing. When $-\phi_0$ reaches $\pi/4$, $M_x = 0$ and the resultant of the applied moments M acts about the beam's minor principal axis, and in its weakest plane.

3.4 Large Twist Rotations

Closed form solutions of Equation 5 for large twist rotations are unknown, but approximate solutions may be obtained by using the limited Taylor series expansion

$$\phi = \phi_0 + \phi_0^{ii} \frac{z^2}{2} + \phi_0^{iv} \frac{z^4}{24} + \phi_0^{vi} \frac{z^6}{720} + \phi_0^{viii} \frac{z^8}{40320} \quad (15)$$

in which

$$\phi_0^{ii} = B - \frac{GJ}{(GJ - M\beta_y)} \frac{8(\phi_{i0} - \phi_{e0})}{L^2} \quad (16a)$$

$$B = a_1^2 \cos \phi_0 - a_2^2 \sin \phi_0 \quad (16b)$$

$$\phi_0^{iv} = -B(a_1^2 \sin \phi_0 + a_2^2 \sin \phi_0) \quad (16c)$$

$$\phi_0^{vi} = (\phi_0^{iv})^2 / B \quad (16d)$$

$$\phi_0^{viii} = \phi_0^{vi} \phi_0^{iv} / B - 30B^2 \phi_0^{iv} \quad (16e)$$

The boundary condition $\phi_{L/2} = 0$ requires that

$$0 = \phi_0 + \phi_0^{ii} \frac{L^2}{8} + \phi_0^{iv} \frac{L^4}{384} + \phi_0^{vi} \frac{L^6}{46080} + \phi_0^{viii} \frac{L^8}{10321020} \quad (17)$$

Equation 17 is a non-linear equation which relates the moment M to the twist rotations ϕ_0 . It can be solved iteratively by trial and error. The twist rotation solutions for no initial twist ($\phi_{i0} = 0$) and no continuous torque ($\phi_{e0} = 0$) shown in Fig. 4 are less than the small rotation solutions. Also shown in Fig. 4 are the moments M_x , M_y obtained from Equations 7. The deviations of these from the values of M are significantly less than the small rotation solutions.

4 BEAM STRENGTH

4.1 Fully Plastic Moment Combinations

For this paper, it is assumed that

$$\frac{b}{t} \sqrt{\frac{f_y}{250}} \leq 14 \quad (18)$$

in which the yield stress f_y is in MPa, in which case the beam is compact according to the local buckling recommendations in Trahair (2002a).

For compact beams, it may be assumed that the beam fails when the principal axis moments at midspan M_x , M_y cause the section to become fully plastic. The residual plastic capacity ratio C can be expressed as (Trahair, 2002a)

$$C = 1 - M_y / M_{py} - (M_x / M_{px})^2 \quad (19)$$

in which the principal axis full plastic moments M_{px} , M_{py} are given by

$$M_{px} = 2 M_{py} = f_y b^2 t / \sqrt{2} \quad (20)$$

The section becomes fully plastic when the residual plastic capacity is exhausted, so that

$$C = 0 \quad (21)$$

4.2 Strength of Example Beam

The variations of the residual plastic capacity ratio C with the applied moments M acting on the example beam defined in Fig. 3 are shown in Fig. 4. $C = 0$ when $M = 5.84E6$ Nmm according to the small rotation theory, and when $M = 6.78E6$ Nmm according to the approximate large rotation theory.

4.3 Effects of Span Length and Beam Attitude on Strength

The effects of span length L and beam attitude on the small and large rotation strength predictions for the beam whose section properties are given in Fig. 3 (and $\phi_{i0} = \phi_{e0} = 0$) are shown in Fig. 5. In this figure, the span length L is plotted non-dimensionally using a modified slenderness

$$\lambda_L = \sqrt{(M_p / M_L)} \quad (22)$$

in which M_L is the limiting moment given by Equation 12 and M_p is given by

$$M_p = (1 - \sqrt{2}/2) f_y b^2 t \quad (23)$$

which is the value of $M_x = M_y = M$ which satisfies $C=0$. For the beam of Fig. 3, $M_p = 7.93$ kNm.

Two small rotation theory predictions for M / M_p are shown, which decrease from 1 as the modified slenderness λ_L increases and approach M_L / M_p . The predictions are lower when the horizontal flange is down (positive β_y) than when it is up (negative β_y).

Two large rotation theory predictions for M / M_p are also shown in Fig. 5. These are significantly higher than the small rotation predictions, especially for large values of the modified slenderness λ_L , for which the small rotation values are less than the dimensionless limiting moment M_L / M_p and much less than the minimum strength value of $M / M_p = M_{py} / \sqrt{2} M_p \approx 0.854$, which corresponds to the central section of the beam having rotated through $-\pi/4$, so that the resultant applied moment $\sqrt{2}M$ acts about the minor principal axis.

4.4 Effects of Initial twist and Load Eccentricity

The initial twist rotations ϕ_{i0} are assumed to vary according to

$$\phi_{i0} = -0.2\sqrt{M_{pxm}/M_{yz}} \quad (24)$$

which are somewhat similar to those assumed in Trahair (2004) for predicting the design strengths of single angle beams that fail by lateral buckling (Trahair, 2003). These negative initial twist rotations increase the negative twist rotations of untwisted beams and reduce the beam strengths. The predicted large rotation strength reductions for the beam whose section properties are given in Fig. 3 are shown in Fig. 6.

The additional effects caused by twist rotations $\phi_{e0} = \sqrt{2Mb} / 2GJ$ due to eccentric loads which act through the flange centre at $e = b/2$ are also shown in Fig. 6. When the flange is up (negative β_y), eccentricity causes negative twist rotations which further add to those due to negative initial twist rotations so that the large rotation strengths are further reduced. When the flange is down (positive β_y), eccentricity causes positive twist rotations which reduce those due to negative initial twist rotations so that the large rotation strengths are increased. Goh, Dayawansa, and Bennetts (1991) also found that eccentric loading increases the strength of a beam whose flange is down, as did Trahair (2007).

5 DESIGN

5.1 Australian Institute of Steel Construction

In their third report (1970b), Leigh and Lay developed working stress design tables for equal angle beams with uniformly distributed loads from their previous research on beams in uniform bending. There are some approximations in these because the non-uniform bending caused by the distributed loads was assumed to have the same effect as uniform bending and because the effect of load height on lateral buckling was ignored. The tables include values for loads acting through the shear centre and for loads at eccentricities $\pm b/2$ equal to half the angle leg length b . The values given for the different eccentricities are very close. Only tables for angles with the horizontal leg up are given, but it is indicated that these can be used for angles with the horizontal leg down provided the sense of the eccentricity is changed (from inside the shear centre to outside, and vice versa). These tables were used as the basis for the equal angle safe working load tables of the Australian Institute of Steel Construction (1987).

The research of Goh, Dayawansa, and Bennetts (1991) was used as the basis for the equal angle limit states design capacity tables of the Australian Institute of Steel Construction (1999).

5.2 American Institute of Steel Construction

The American Institute of Steel Construction's Specification (AISC, 2005a) provides rules for designing equal angle beams which are bent in the plane of one leg. These are based on an adaptation of the predictions of Equation 12 for the limiting moment, and a reduction of these to allow for geometrical imperfections and residual stresses.

The adaptation of Equation 12 for the limiting moment M_L substitutes the overall leg length $(b + t / 2)$ for the thin-walled leg length b , which is unconservative, and uses a value of $E/G = (29000 \text{ ksi}) / (11200 \text{ ksi}) \approx 2.59$ instead of $(200000 \text{ MPa}) / (80000 \text{ MPa}) = 2.5$ used in this paper.

The reduction used to allow for geometrical imperfections and residual stresses has the same form as that used to reduce the elastic lateral buckling moments of beams. However, the logic of this can be questioned since the elastic buckling moment can be closely approached, whereas the limiting moment M_L which corresponds to infinitely large twist rotations cannot. For the reduction, the maximum nominal strength is taken as

$$\sqrt{2}M_{nm} = 1.2 f_y Z_X \quad (25)$$

in which the rectangular axis elastic section modulus is given by (Bridge and Trahair, 1981)

$$Z_X = 5 b^2 t / 18 \quad (26)$$

This value of $\sqrt{2}M_{nm}$ is close to the $0.8 \times \sqrt{2}M_p$, and so is quite conservative. In order to make the comparisons of this paper, the reduction has been applied to the limiting moments of Equation 12 which are based on the thin-walled leg length b and $E/G = 2.5$. Thus the "AISC" nominal strengths are given by

$$\begin{aligned} \sqrt{2}M_n &= \sqrt{2}M_{nm} & \text{while } 0 \leq M_{nm} / 1.5M_L < 0.129 \\ \sqrt{2}M_n &= \sqrt{2}M_{nm} (1.28 - 0.78 \sqrt{M_{nm} / 1.5M_L}) & \text{while } 0.129 \leq M_{nm} / 1.5M_L \leq 1 \\ \sqrt{2}M_n &= \sqrt{2}M_L (0.92 - 0.17 / (M_{nm} / 1.5M_L)) & \text{while } 1 \leq M_{nm} / 1.5M_L \end{aligned} \quad (27)$$

These nominal strengths are shown in Fig. 5. It can be seen that while they approach the small rotation strength predictions at high slenderness, they are significantly less than the large rotation predictions.

5.3 Proposed Nominal Design Strengths

Perhaps the simplest proposal for the nominal design strength of any simply supported equal single angle steel lintel beam is to use

$$\sqrt{2}M_n = 0.85M_p \quad (27)$$

A somewhat more economical method is to use

$$\begin{aligned}
 \sqrt{2}M_n &= \sqrt{2}M_p & \text{while } 0 \leq \lambda_L \leq \lambda_{Lx} \\
 \sqrt{2}M_n &= \sqrt{2}M_p (k_0 - k_L \lambda_L) & \text{while } \lambda_{Lx} \leq \lambda_L \leq \lambda_{Ly} \\
 \sqrt{2}M_n &= 0.85 \times \sqrt{2}M_p & \text{while } \lambda_{Ly} \leq \lambda_L
 \end{aligned} \tag{28}$$

in which the values of λ_{Lx} , λ_{Ly} , k_0 , and k_L are given in Table 1 and M_p is given by Equation 23. These approximations are shown in Fig. 7a.

Flange	Initial Twist	Eccentricity	λ_{Lx}	λ_{Ly}	k_0	k_L
Up	Yes	No	0.4	1.0	1.1	0.25
Up	Yes	Yes	0.2	0.95	1.04	0.2
Down	Yes	No	0	0.75	1.0	0.2
Down	Yes	Yes	0.2	0.8	1.05	0.25

This method will be a little conservative for lintels with other loading conditions, such as uniformly distributed load q or central concentrated load Q .

5.4 Serviceability Design

Because the strength design recommendations above are significantly higher than those of the AISC (2005a), smaller lintels may be required, in which case serviceability considerations will become more important, even though serviceability design loads are usually significantly less than strength loads.

Serviceability limits for twist rotations are difficult to formulate, but deflection limits are in common use. It is therefore suggested that the serviceability of lintels should be assessed by comparing their deflections with acceptable deflection limits.

The calculation of serviceability deflections is traditionally not as exact as strengths, partly because serviceability limits are not as closely defined as strength limits. Consequently, only a very simple approximate method of calculating serviceability deflections is proposed.

This may be based on the linear interpolations shown in Fig. 7b between the most optimistic δ_o and the most pessimistic δ_p of predictions of the maximum deflection in the plane of bending, according to

$$\delta = \delta_o + (\delta_p - \delta_o) \lambda_L / \lambda_{Ly} \leq \delta_p \tag{29}$$

in which λ_L is the modified slenderness of Equation 22 and values of λ_{Ly} are given in Table 1.

The most optimistic prediction δ_o is obtained by ignoring all non-linearities and using the linear elastic prediction (see Appendix 1) of

$$\delta_o = 2.5(\sqrt{2}ML^2 / 8EI_x) \quad (30)$$

for a lintel in uniform bending. An approximation for the most pessimistic prediction δ_p is obtained in Appendix 1 as

$$\delta_p = 3.9(\sqrt{2}ML^2 / 8EI_x) \quad (31)$$

These equations may be conservatively applied to lintels with uniformly distributed load q or central concentrated load Q by substituting $5qL^4/384EI_x$ or $QL^3/48EI_x$ respectively for $\sqrt{2}ML^2/8EI_x$.

6. CONCLUSIONS

This paper reviews past research on unrestrained compact single equal angle steel beams used as lintels, and develops an improved method of predicting their principal axis bending moments which includes the effects of eccentric loads, lateral buckling influences, monosymmetry, initial twist rotations, and large twist rotations. This method is used with formulations of the full plastic moment capacities of compact beams to predict their nominal strengths.

Monosymmetry of the equal angle section causes a concentrically loaded lintel to rotate so that the applied loading acts more nearly in its weakest plane, thereby decreasing its strength. The rotations and strengths of lintels with the horizontal flange up are less than those with the flange down.

Small rotation elastic analysis significantly overestimates the twist rotations and principal axis moments and underestimates the strength. Large rotation analysis predicts that the strength is never less than that for bending in the weakest plane. Initial twist decreases the strength, and so does eccentric loading when the flange is up, but eccentric loading increases the strength when the flange is down.

The design basis of the AISC (2005a, b) ignores initial twist and eccentricity, overestimates the effective leg lengths of equal angles, and underestimates the plastic moment capacities. The design strengths are based on small rotation theory, and are very conservative. The improved strength approximations developed in this paper will lead to significant economies.

A simple serviceability design method is suggested.

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APPENDIX 2 NOTATION

A	area of cross-section
a_1, a_2	see Equations 10
b	leg length
C	residual plastic capacity ratio
E	Young's modulus of elasticity
e	eccentricity of load from the shear centre
f_y	yield stress
G	shear modulus of elasticity
I_e	effective second moment of area in plane of bending
I_n	"Wagner" section constant
I_x, I_y	second moments of area about the x, y principal axes
J	torsion section constant
k_0, k_L	constants in Equation 28 (see Table 1)
L	span length
M	applied end moments
M_L	limiting value of M
M_n	nominal moment strength
M_{nm}	maximum nominal moment strength (see Equation 25)
M_p	value of M at full plasticity
M_{pxm}, M_{pym}	fully plastic moments about the x, y axes
M_x, M_y	moments about the x, y principal axes
M_{yz}	elastic buckling moment
M_z	torque
m	intensity of uniformly distributed torque
Q	central concentrated load
q	intensity of uniformly distributed load
t	leg thickness
u, v	deflections in the x, y directions
X, Y	rectangular (geometric) axes
x, y	principal axes
x_0	shear centre distance
Z_X	elastic section modulus about X axis
z	distance along beam
α	angle between x axis and applied moment
β_y	monosymmetry section constant
δ	serviceability deflection
δ_o	optimistic value of δ
δ_p	pessimistic value of δ
λ_L	modified slenderness
$\lambda_{Lx}, \lambda_{Ly}$	constants in Equation 28 (see Table 1)
ϕ	twist rotation
ϕ_{e0}	central twist rotation caused by distributed torque
ϕ_i	initial twist rotation
ϕ_{i0}	central initial twist rotation
ϕ_0	central twist rotation

APPENDIX 3 SERVICEABILITY DEFLECTIONS

The maximum elastic deflection of an equal angle beam depends on the plane of bending. The moment $\sqrt{2}M$ shown in Fig. 8 acts at an angle

$$\alpha = \pi / 4 + \phi \quad (32)$$

to the x principal axis. The maximum principal plane deflections are given by

$$\begin{aligned} u_0 &= -\sqrt{2}ML^2 \sin \alpha / 8EI_y \\ v_0 &= +\sqrt{2}ML^2 \cos \alpha / 8EI_x \end{aligned} \quad (33)$$

so that the maximum deflection perpendicular to the applied moment is given by

$$\delta_0 = (\sqrt{2}ML^2 / 8) \{ \sin^2 \alpha / EI_y + \cos^2 \alpha / EI_x \} \quad (34)$$

which becomes

$$\delta_0 = \{4 - 3 \cos^2 \alpha\} (\sqrt{2}ML^2 / 8EI_x) \quad (35)$$

when $I_y = I_x / 4$ is used. This can be used to express the effective second moment of area as

$$I_e = I_x / \{4 - 3 \cos^2 \alpha\} \quad (36)$$

which can be approximated by

$$I_e = I_x / \{4 - 1.5(1 - 4\phi/\pi)^2\} \quad \text{when } 0 \leq \phi \leq \pi/4 \quad (37)$$

The most optimistic prediction δ_o of a lintel beam is obtained by ignoring any twist rotation ϕ , in which case $\alpha = \pi/4$ and

$$\delta_o = 2.5(\sqrt{2}ML^2 / 8EI_x) \quad (38)$$

The most pessimistic prediction δ_p may be obtained by noting that the worst strength condition occurs when the central cross-section of the lintel has rotated through $\phi = \pi / 4$, so that the applied moment acts in the weakest plane. In this case, the effective second moment of area I_e of the lintel will vary from $I_x / 2.5$ at the support to $I_x / 4$ at mid span. If it is assumed that the twist rotation is approximated by

$$\phi = \pi\{1 - (2z/L)^2\} / 4 \quad (39)$$

then the differential equation of bending is

$$-EI_x \delta'' = \sqrt{2}M\{4 - 1.5(2z/L)^4\} \quad (40)$$

The solution of this which satisfies the boundary conditions $\delta_{L/2} = 0$, $\delta'_0 = 0$ is

$$-EI_x \delta = \sqrt{2}M\{2z^2 - 4z^6/5L^4 - 39L^2/80\} \quad (41)$$

so that the central deflection is

$$\delta_p = 3.9(\sqrt{2}ML^2 / 8EI_x) \quad (42)$$

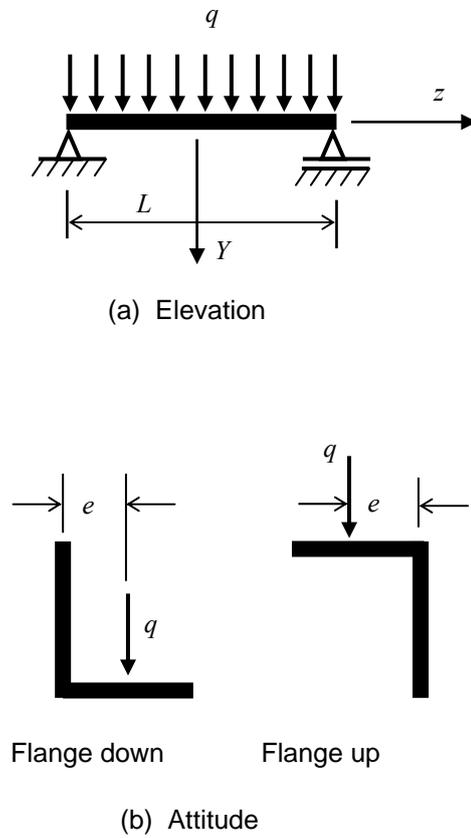


Fig. 1. Uniformly Loaded Equal Angle Lintel Beam

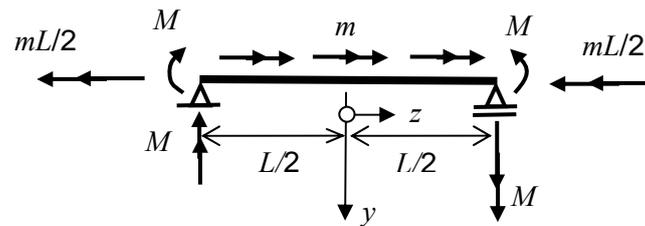
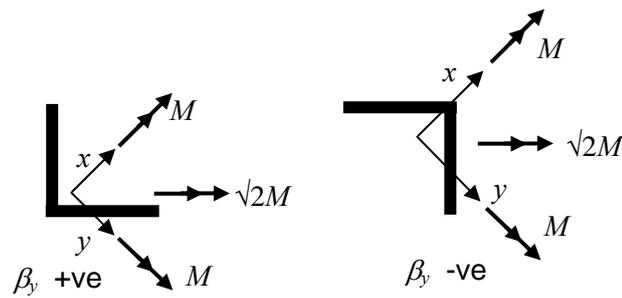
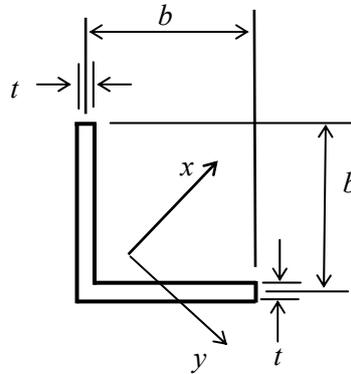
(a) Elevation on yz plane(b) Cross sections at $z = L/2$

Fig. 2. Equal Angle Lintel Beams in Uniform Bending and Torsion



(a) Cross-section

$b = 95 \text{ mm}$	$f_y = 300 \text{ MPa}$
$t = 10 \text{ mm}$	$E = 2 \text{ E5 MPa}$
$I_x = b^3 t / 3 = 2.858 \text{ E6 mm}^4$	$G = 8 \text{ E4 MPa}$
$I_y = b^3 t / 12 = 0.7145 \text{ E6 mm}^4$	$M_{pxm} = (\sqrt{2}/2) f_y b^2 t = 19.14 \text{ E6 Nmm}$
$J = 2 b t^3 / 3 = 0.06333 \text{ E6 mm}^4$	$M_{pym} = (\sqrt{2}/4) f_y b^2 t = 9.572 \text{ E6 Nmm}$
$\beta_y = \sqrt{2} b = 134.4 \text{ mm}$	$M_p = (1 - \sqrt{2}/2) f_y b^2 t = 7.930 \text{ E6 Nmm}$

(b) Section properties

Fig. 3 Example: Equal Angle Section

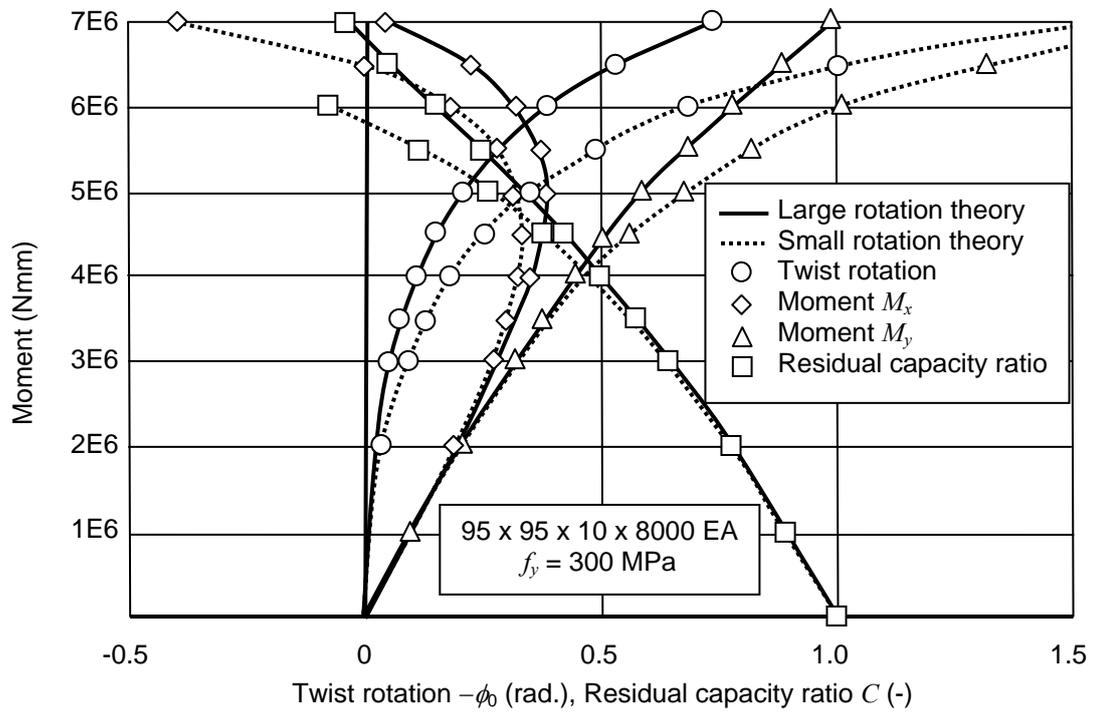


Fig. 4 Small and Large Twist Rotation Effects

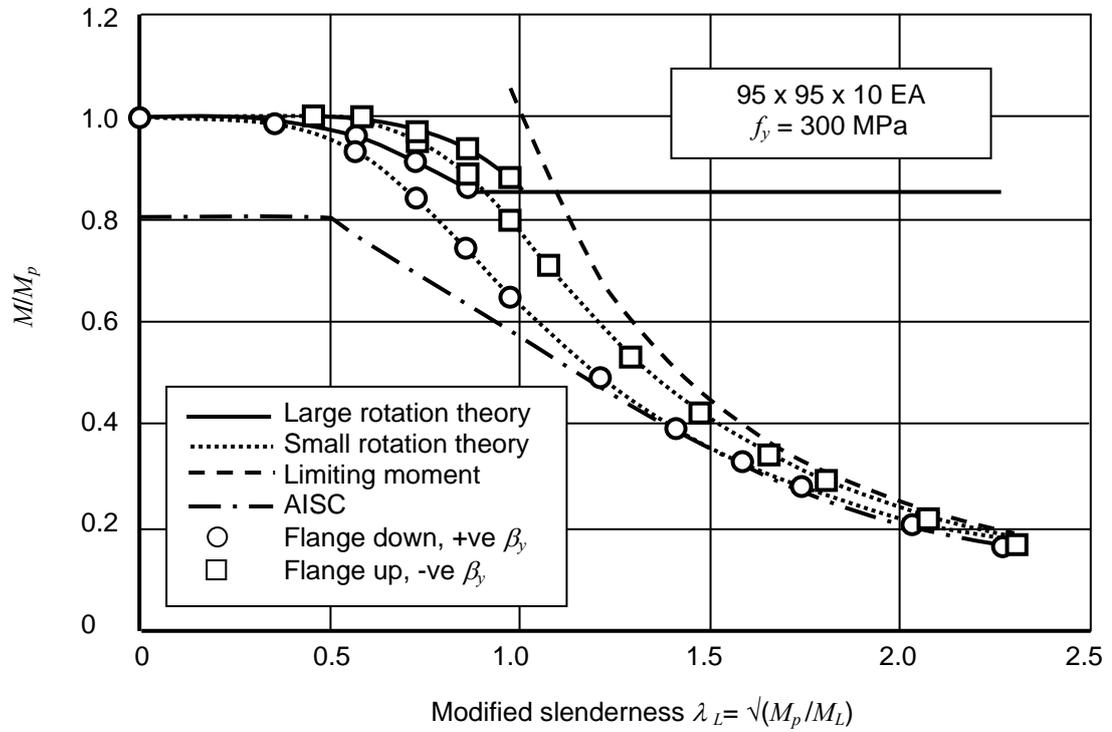


Fig. 5 Slenderness and Attitude Effects

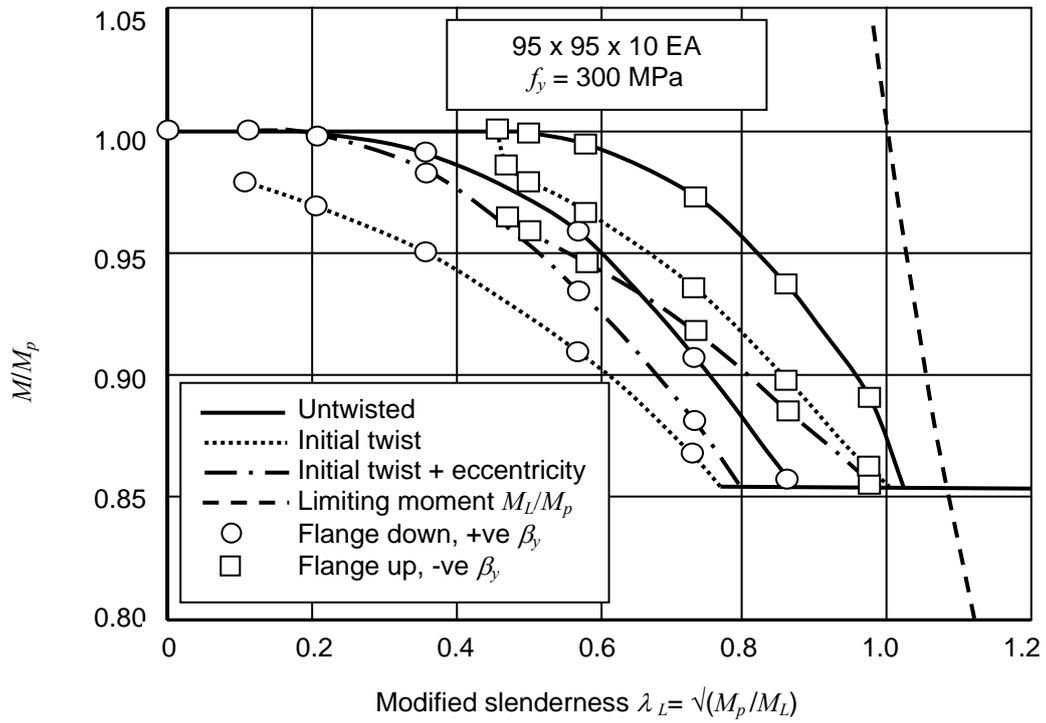
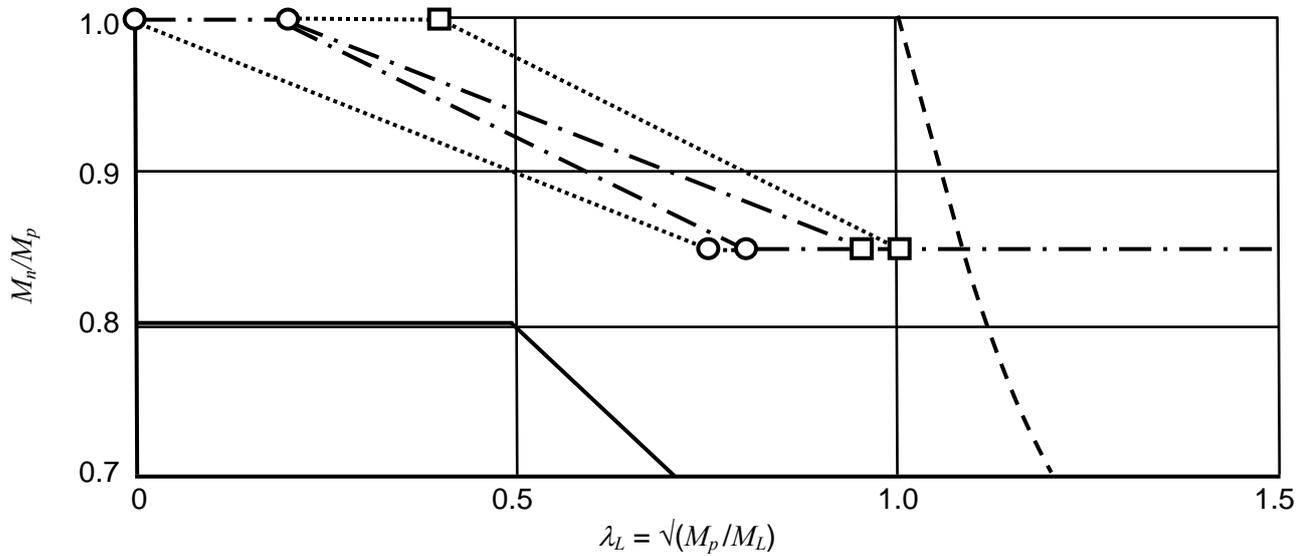
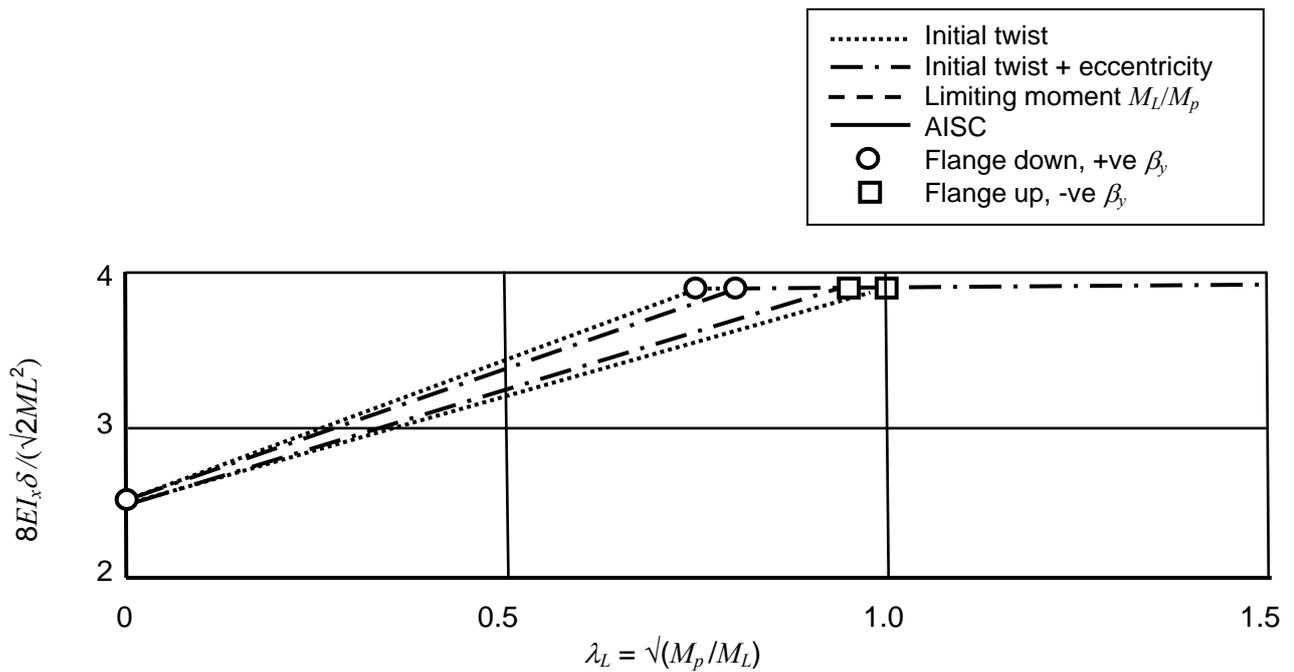


Fig. 6 Initial Twist and Eccentricity Effects



(a) Nominal strengths



(a) Serviceability deflections

Fig. 7 Design Proposals

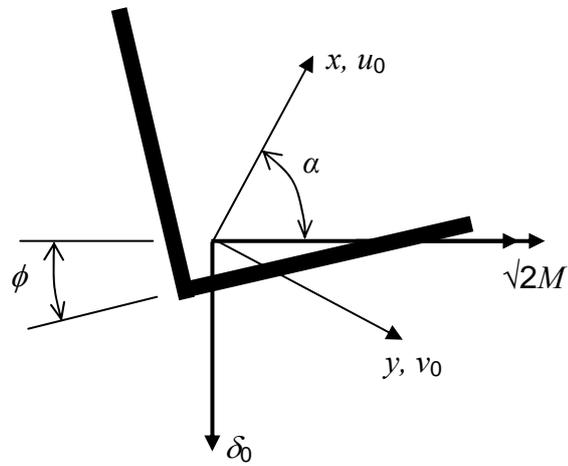


Fig. 8 Serviceability deflection