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Research Report No R891

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Abstract:

Steel design codes do not provide sufficient information for the efficient design of steel structures against out-of-plane failure, and what is provided is often overly conservative. The method of design by buckling analysis corrects this situation for beams, but the extension of this method to columns is only suggested, while there is no guidance on how to apply this method to the design of beam-columns and frames.

Beam design by buckling analysis uses the design code formulation for the member nominal design strengths in terms of the section moment capacities and the maximum moments at elastic buckling, accurate predictions of which may be determined by available computer programs. Column design by buckling analysis is similar to beam design, in that it uses the design code formulation for the column nominal design strengths in terms of the section compression capacities and accurate predictions of the elastic buckling loads which may also be obtained from computer programs.

However, design codes do not provide formulations for the direct buckling design of beam-columns, but instead use the separate results of beam design and column design in interaction equations. The further extension to frames is not directly possible, because frames are not designed as a whole (except through the rarely used methods of advanced analysis), but as a series of individual members. This paper shows how the method of design by buckling analysis can be used to design beam-columns and frames as well as beams and columns. Two example frames are designed and very significant economies are demonstrated when the method of design by buckling analysis is used.

Keywords: beams, beam-columns, bending, buckling, columns, compression, design, frames, member strength, moments, steel.

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1 INTRODUCTION

Steel beam design by out-of-plane (flexural-torsional) buckling analysis allows greater efficiency than when the buckling approximations embedded in some design codes such as AS4100 [1] or BS5950 [2] are used. Design by buckling analysis is allowed in AS4100 and implied in EC3 [3]. Steel column design by buckling analysis is suggested in the Commentary to AS4100 [4] and implied in EC3. However, no methods are given or suggested for designing beam-columns or frames against out-of-plane failure.

Beam design by buckling analysis uses the design code formulation for the member nominal design strength M_{bx} in terms of the section capacity M_{sx} and the elastic buckling moment M_{oo} of the basic simply supported beam in uniform bending. Column design by buckling analysis is similar to beam design, in that it uses the design code formulation for the column nominal design strength N_{cy} in terms of the section capacity N_s and the elastic buckling load N_{oc} of the basic simply supported column. For other than the basic cases, the elastic out-of-plane buckling moment or load needs to be determined.

Elastic out-of plane buckling of beams and columns [5] depends on:

- (a) the elastic moduli, the section properties, and the length,
- (b) the distribution of the moments and compressions and the load heights,
- (c) the restraints,
- (d) any non-uniformity of the section, and
- (e) interactions between these.

The effects of the elastic moduli, the section properties, and the length are allowed for in the basic cases, and formulations are given in most [1, 2, 6] but not all [3] codes. The effect of the distribution of moments is allowed for approximately in most codes [1, 2, 3, 6]. The effect of load height is partially allowed for in some codes [1, 2], using fairly inaccurate approximations.

Restraints may be concentrated or distributed, and may be elastic or rigid. Restraints may be translational, rotational (out-of-plane), torsional, or warping. The effects of translational and rotational restraints depend on their height. No codes allow for the full range of restraint effects. Some [1, 2] allow approximately for partial torsional end restraints, and some [1, 2, 6] allow for rotational elastic end restraints acting at the centroid.

Some design codes [1, 2, 6] provide limited approximations for the effect of non-uniformity of the section.

Design codes assume that these effects are largely independent so that they can be treated separately, but this is not the case [7]. Thus there is a lack of precision in the often conservative approximations used in codes to determine the elastic buckling moments and loads. This can be overcome to a certain extent by using some of the wealth of published research information [7] but this is difficult to access. However, for some time now computer programs [8, 9] have been available, and some of these such as ABAQUS [10] are often used by designers. The use of these programs now provides a viable method of carrying out the efficient and economical design of beams and columns by buckling analysis.

However, the extension of design by buckling analysis to beam-columns cannot be carried out in the same way, even though accurate predictions of the elastic buckling loads can be obtained. The reason for this is that design codes do not provide formulations for the buckling design of beam-columns, but instead use the separate results of beam design and column design in interaction equations. The further extension to frames is also not possible, because frames are not designed as a whole (except through the rarely used methods of advanced analysis [11]) but as a series of individual members.

The purposes of this paper are to show how the methods of design of beams and columns by buckling analysis can be used to design beam-columns and frames.

2 DESIGN BY BUCKLING ANALYSIS

2.1 Beams

The basic design code case for beams is the simply supported doubly symmetric beam in uniform bending, for which the moment which causes elastic flexural-torsional buckling [7] is

$$M_{oo} = \sqrt{\frac{\pi^2 EI_y}{L^2}\left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \tag{1}$$

in which E and G are the Young's and shear moduli of elasticity, I_y, J and I_w are the minor axis second moment of area, torsion and warping section constants, and L is the length.

Most design codes provide approximations which allow for the effect of non-uniform bending in simply supported beams loaded through the shear centre through formulations of the type

$$M_{os} = \alpha_m M_{oo} \tag{2}$$

in which the moment modification factor α_m is approximated by [1]

$$\alpha_m = \frac{1.7 M_m}{\sqrt{(M_2^2 + M_3^2 + M_4^2)}} \leq 2.5 \tag{3}$$

in which M_m is the maximum moment and M_2, M_3, and M_4 are the moments at the quarter-, mid-, and three-quarter points, or by similar expressions [2, 6]. Alternatively, α_m may be determined from Equation 2 by using the value of M_{os} determined by an elastic buckling analysis. The effects of load height and end restraints are allowed for approximately in some codes [1, 2] by replacing the beam length L in Equation 1 with an effective length L_e.

The method of design by buckling analysis of the Australian code AS4100 [1] allows the direct use of the results of elastic buckling analyses. For this, the maximum moment M_{ob} at elastic buckling is used in the equation

$$\frac{M_{bx}}{M_{sx}} = 0.6\alpha_m \left\{ \sqrt{\left[\left(\frac{\alpha_m M_{sx}}{M_{ob}} \right)^2 + 3 \right]} - \left(\frac{\alpha_m M_{sx}}{M_{ob}} \right) \right\} \leq 1 \tag{4}$$

to determine the nominal major axis moment strength M_{bx}, in which M_{sx} is the nominal major axis section capacity (reduced below the full plastic moment M_{px} if necessary to allow for local buckling effects). The variations of the dimensionless nominal strength M_{bx} / M_{sx} with the modified beam slenderness $\lambda_b = \sqrt{(M_{sx} / M_{ob})}$ and the moment modification factor α_m are shown in Fig. 1. It can be seen that as the value of α_m increases, the nominal design strengths M_{bx} approach the elastic buckling moments M_{ob}, reflecting the additional influence of non-uniform bending on inelastic buckling [5, 7, 12].

2.2 Columns

The basic design code case for columns is the simply supported column in uniform compression, for which the load which causes elastic flexural buckling is

$$N_{oc} = \frac{\pi^2 E I_y}{L^2} \tag{5}$$

Many design codes [1, 2, 6] provide approximations which allow for the effect of flexural end restraints on simply supported columns by replacing the column length L with an effective length L_e. Some codes for hot-rolled steel structures do not allow for column failure by flexural-torsional buckling [7, 13].

The method of design by buckling analysis of the Australian code AS4100 [1, 4] allows the direct use of the results of elastic buckling analyses. For this paper, the AS4100 nominal design strength N_{cy} can be approximated by using

$$N_{cy} / N_s = 1.003 + 0.095\lambda_c - 0.832\lambda_c^2 + 0.409\lambda_c^3 - 0.058\lambda_c^4 \leq 1 \tag{6}$$

in which N_s is the nominal section capacity (reduced below the full plastic load Af_y if necessary to allow for local buckling effects), $\lambda_c = \sqrt{(N_s / N_{om})}$ is the modified column slenderness and N_{om} is the elastic buckling load, although a more complicated general formulation is given in the code. The variations of this dimensionless nominal strength N_{cy} / N_s with the modified slenderness λ_c are shown in Fig. 2.

2.3 Beam-Columns and Frames

Design codes do not explicitly allow the use of a method of design by buckling analysis for the out-of-plane design of beam-columns and frames. Instead, each member of a frame is considered as a beam-column and designed independently by using out-of-plane interaction equations of the type

$$\frac{N_{max}}{N_{cy}} + \frac{M_{max}}{M_{bx}} = 1 \tag{7}$$

in which N_{max} and M_{max} are the maximum nominal design actions (which are often reduced by using capacity (or resistance) factors). Thus each beam-column is considered first as a beam to determine M_{bx} and second as a column to determine N_{cy}, before these are used in Equation 7.

When buckling analyses are used in the determination of M_{bx} and N_{cy}, then this becomes the method of design by buckling analysis. This method is demonstrated in the following sections for the two example frames shown in Figs 3 and 4, and compared with the use of the normal code method of design without buckling analysis. For these demonstrations and comparisons, the Australian design code AS4100 [1] is used, but they could equally well be done by using other codes [2, 3, 6].

3 EXAMPLE FRAME 1

The members of the pin-based portal frame shown in Fig. 3a have the properties shown in Fig. 5. The horizontal member has two equal transverse loads (initially equal to 1) acting at the bottom flange. Warping is prevented at both ends and there are elastic translational restraints of stiffness α_t = 10 N/mm acting at the load points at the bottom flange. The in-plane reactions and moment distribution are shown in Fig. 3b.

For this frame, the design is controlled by the horizontal member. For this, this member is first treated as the beam shown in Fig. 3c, and then as the column shown in Fig. 3d. The results of these are then used in Equation 7.

The results (No DBA) of using the Australian code AS4100 [1] alone are summarized in Table 1. AS4100 does not provide any guidance on warping restraint [7, 14, 15], while the bottom flange loads and elastic translational restraints cannot be accounted for. Because of this, the values determined for the maximum nominal design actions N_{max} and M_{max} are quite conservative.

Also summarized in Table 1 are the results (DBA) of using the method of design by buckling analysis. For this the elastic buckling moments M_{oo}, M_{os}, M_{ob} were determined using the computer program PRFELB [8, 9] which is able to account for the warping and translational restraints and the bottom flange loading. It can be seen that the values determined for the maximum nominal design actions N_{max} and M_{max} are significantly higher than those determined without using design by buckling analysis.

Table 1 Elastic Buckling and Design

Quantity	Units	Frame 1 DBA	Frame 1 No DBA	Frame 2 DBA	Frame 2 No DBA
M_{ob}	kNm	61.38	–	107.12	–
M_{os}	kNm	45.79	–	72.12	–
M_{oo}	kNm	25.78	25.78	40.00	40.00
α_m	–	1.776	1.719	1.803	1.817
M_{ob}/α_m	kNm	34.56	25.78	59.42	40.00
M_{sx}	kNm	155.52	155.52	155.52	155.52
α_s	–	0.1931	0.1463	0.3127	0.2210
M_{bx}	kNm	53.34	39.10	87.68	62.45
N/M	–	0.2000	0.2000	1.867	1.867
N_s	kN	1520	1520	1520	1520
N_{om}	kN	93.32	49.66	270.63	111.72
N_{cy}	kN	89.03	48.34	243.59	105.82
M_{max}	kNm	42.87	30.29	47.20	26.74
N_{max}	kN	8.57	6.06	88.11	49.92

The program PRFELB is able to determine the elastic buckling load factors λ_o (i.e. the ratios of values of the actions at elastic buckling to the initial values) of beam-columns and frames, as well as those of beams and columns. While these are not required for design by buckling analysis, they are shown in Table 2. The value of λ_o = 20940 for the beam-column having the combination of the actions shown in Fig. 3c and d is slightly different to that of λ_o = 20990 for the frame. This is because the beam-column is assumed to be prevented from twisting but free to rotate horizontally at both ends, although in the frame end twisting and rotation are elastically restrained by the vertical members.

Table 2 Buckling Load Factors λ_o

	Frame 1	Frame 2
Beam	22520	200000
Column	171200	270600
Beam-Column	20940	128100
Frame	20990	128400

4 EXAMPLE FRAME 2

The members of the pin-based portal frame shown in Fig. 4a have the properties shown in Fig. 5. The horizontal member has a central transverse load (initially equal to 2) acting at the centroid. Warping is prevented at both ends and there are rigid translational and torsional restraints acting at the load point. Each vertical member has an elastic translational restraint of stiffness α_t = 100 N/mm acting at the outer flange. The in-plane reactions and moment distribution are shown in Fig. 4b. For this frame, the design is controlled by the vertical members.

The results (No DBA) of using the Australian code AS4100 [1] alone are summarized in Table 1. AS4100 does not provide any guidance on warping restraint [7, 14, 15], while the outer flange elastic translational restraints cannot be accounted for. Because of this, the values determined for the maximum nominal design actions N_{max} and M_{max} are quite conservative.

Also summarized in Table 1 are the results (DBA) of using the method of design by buckling analysis. It can be seen that the values determined for the maximum nominal design actions N_{max} and M_{max} are significantly higher than those determined without using design by buckling analysis.

The elastic buckling load factors λ_o are shown in Table 2. The value of λ_o = 12810 for the beam-columns having the combination of the actions shown in Fig. 4c and d is slightly different to that of λ_o = 12840 for the frame. This is because the beam-columns are assumed to be prevented from twisting but free to rotate vertically at the top, although in the frame end twisting and rotation are elastically restrained by the horizontal member.

5. CONCLUSIONS

Steel design codes [1, 2, 3, 6] do not provide sufficient information for the efficient design of steel structures against out-of-plane failure, and what is provided is often overly conservative. The method of design by buckling analysis explicitly permitted by the Australian code AS4100 [1] corrects this situation for beams, but the extension of this method to columns is only suggested [1, 4], while there is no guidance on how to apply this method to the design of beam-columns and frames.

Beam design by buckling analysis uses the design code formulation for the member nominal design strengths M_{bx} in terms of the section moment capacities M_{sx} and the maximum moment M_{ob} at elastic buckling, accurate predictions of which may be determined by available computer programs [8, 9, 10]. Column design by buckling analysis is similar to beam design, in that it uses the design code formulation for the column nominal design strengths N_{cy} in terms of the section compression capacities N_s and accurate predictions of the elastic buckling load N_{om} which may also be obtained from computer programs.

However, design codes do not provide formulations for the direct buckling design of beam-columns, but instead use the separate results of beam design and column design in interaction equations. The further direct extension to frames is also not possible, because frames are not designed as a whole (except through the rarely used methods of advanced analysis [11]) but as a series of individual members. This paper shows how the method of design by buckling analysis can be used to design beam-columns and frames as well as beams and columns. Two example frames are designed and very significant economies are demonstrated when the method of design by buckling analysis is used.

APPENDIX 1 REFERENCES

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APPENDIX 2 NOTATION

A	area of cross-section
E	Young's modulus of elasticity
f_y	yield stress
G	shear modulus of elasticity
I_x, I_y	second moments of area about the x, y principal axes
I_w	warping section constant
J	torsion section constant
L	length
L_e	effective length
M	bending moment
M_{bx}	beam moment capacity
M_m	maximum moment in member
M_{max}	maximum nominal design moment
M_{ob}	maximum moment at elastic buckling
M_{oo}	M_{ob} for a simply supported beam in uniform bending
M_{os}	M_{ob} for a simply supported beam with shear centre loading
M_{pxm}	fully plastic moment about the x axis
M_{sx}	section moment capacity
M_2, M_3, M_4	moments at quarter-, mid-, and three quarter-points
N	axial compression
N_{cy}	column compression capacity
N_{max}	maximum nominal design compression
N_{om}	N at elastic buckling
N_{oy}	N_{om} for a simply supported column
x, y	principal axes
α_m	moment modification factor
α_s	slenderness reduction factor
α_t	stiffness of translational restraint
λ_b	modified beam slenderness
λ_c	modified column slenderness
λ_o	buckling load factor

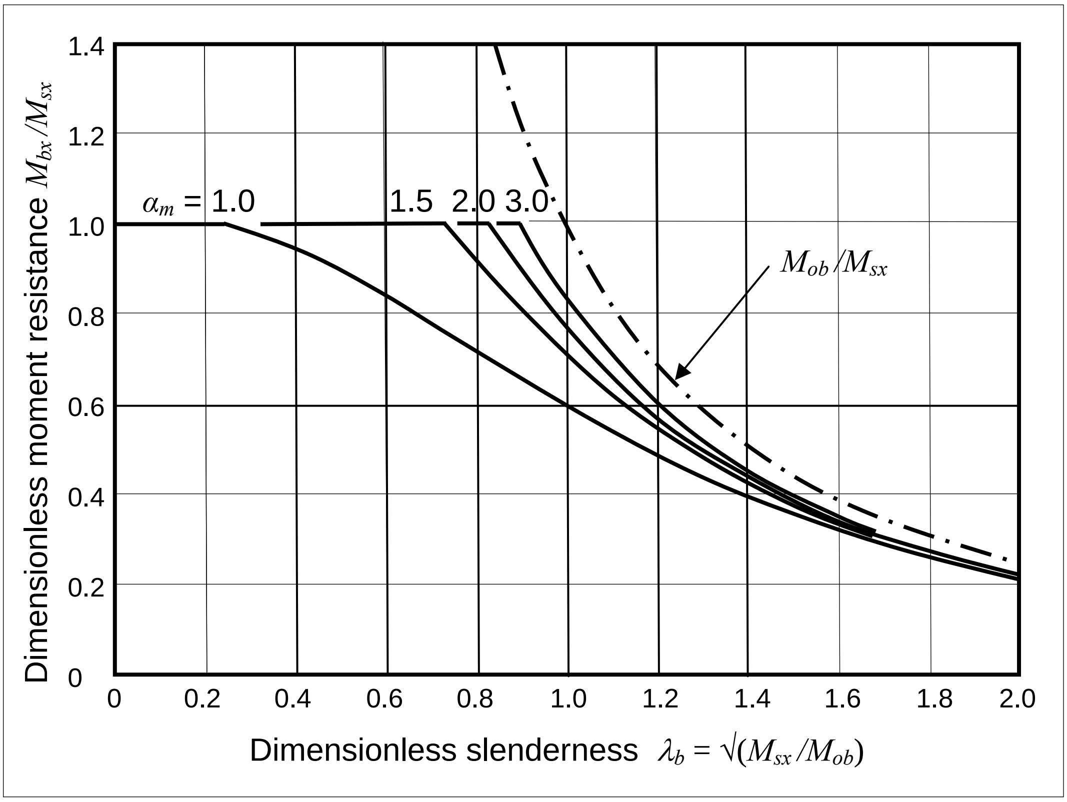


Fig. 1 Beam lateral buckling moment resistances of AS4100

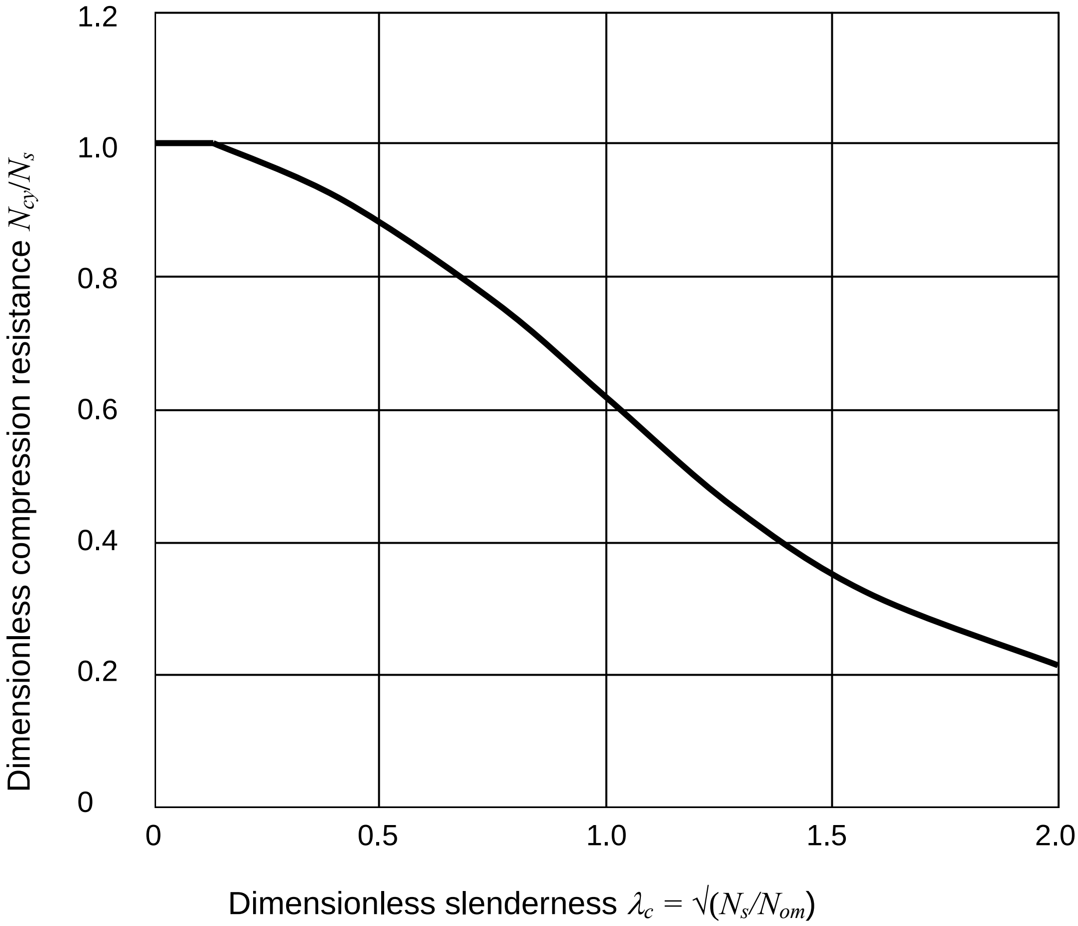


Fig. 2 Column buckling compression resistance of AS4100

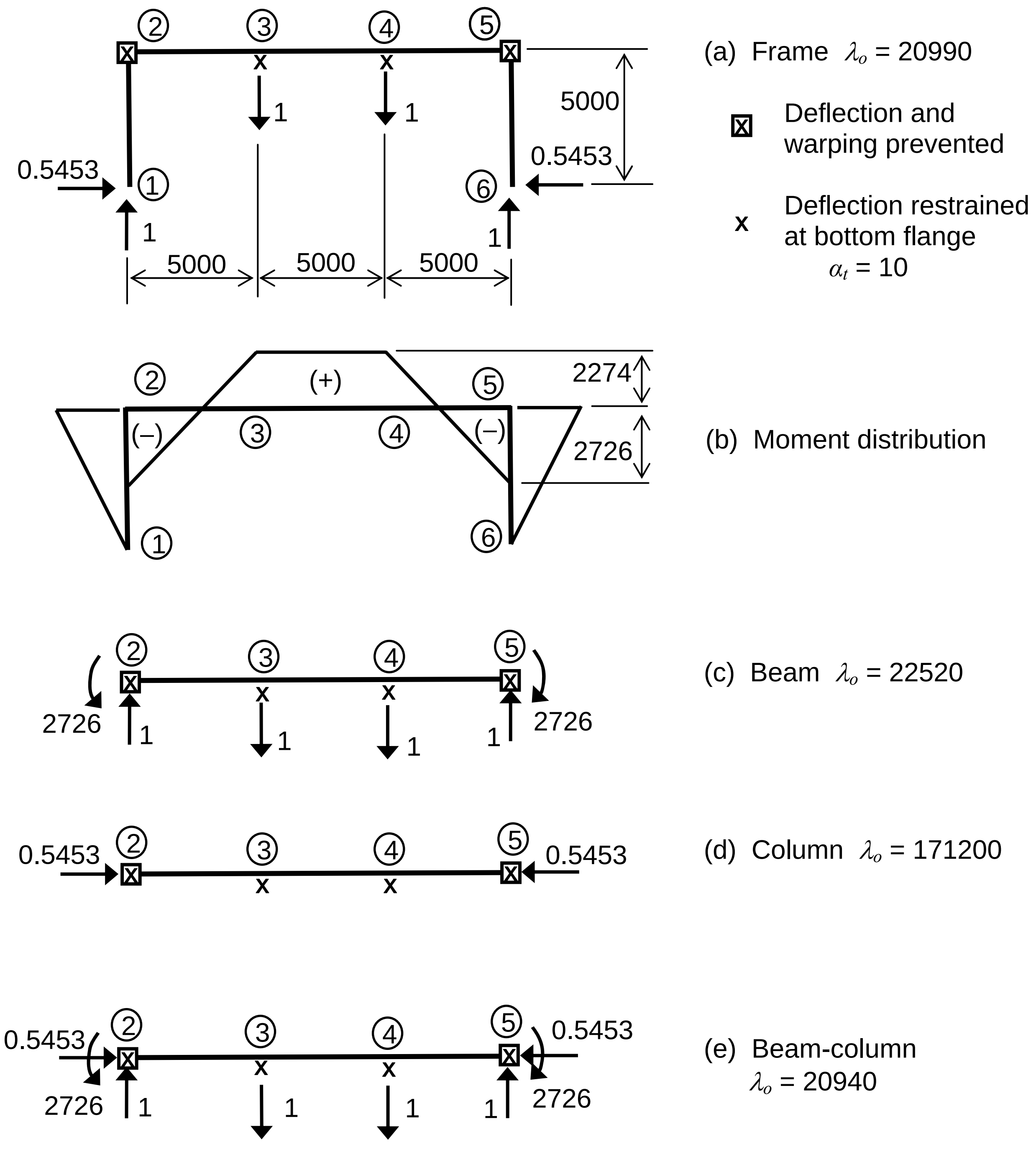
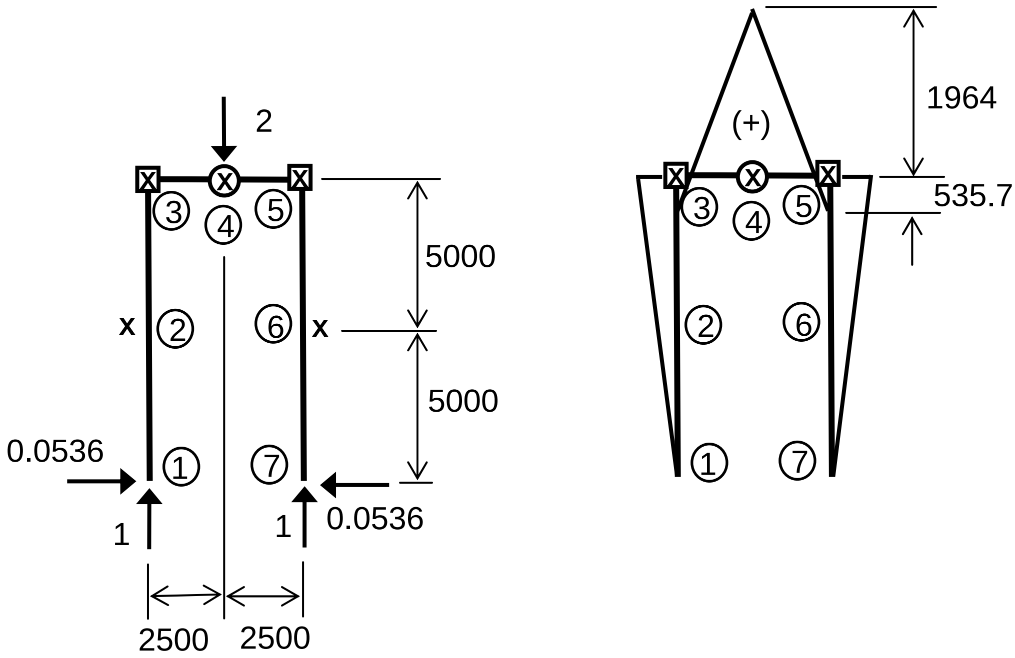


Fig. 3 Example Frame 1

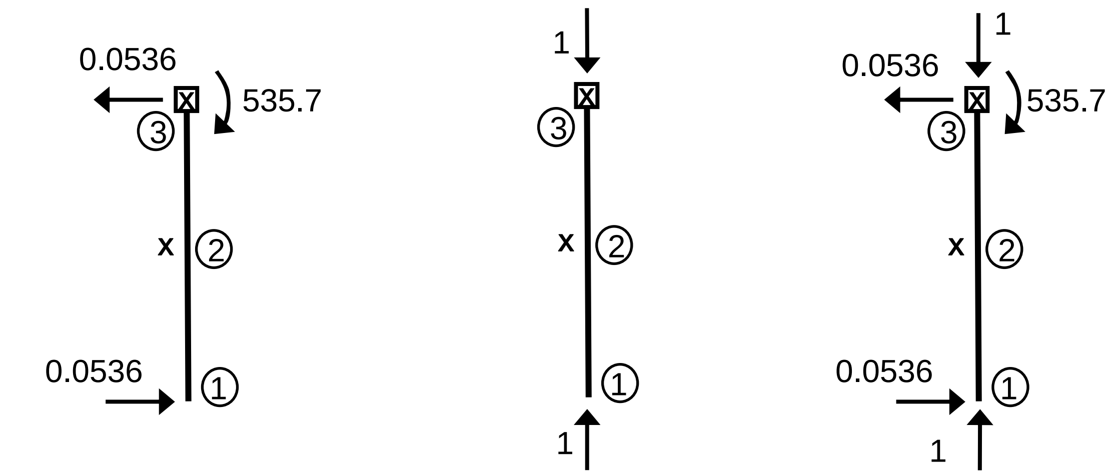


(a) Frame λ_o = 128400 (b) Moment distribution

⊠ Deflection and warping prevented

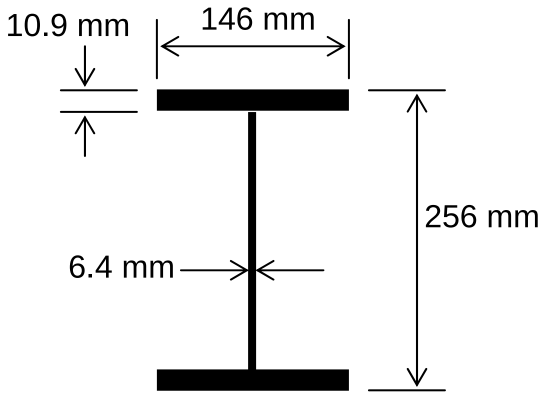
x Deflection restrained at outer flange α_t = 100

⊗ Deflection and twist rotation prevented



(c) Beam λ_o = 200000 (d) Column λ_o = 270600 (e) Beam-column λ_o = 128100

Fig. 4 Example Frame 2



(a) Beam and Dimensions

250 UB 37.3 Grade 300

(b) Material properties

E = 2E5 N/mm^2
G = 8E4 N/mm^2
f_y = 320 N/mm^2

(c) Section Properties

A = 4750 mm^2 I_x = 55.7E6 mm^4
I_y = 5.66E6 mm^4 J = 158E3 mm^4 I_w = 85.2E9 mm^6
N_s = 1520 kN M_{sx} = 155.52 kNm

Fig. 5 Beam Section and Properties