INELASTIC BUCKLING OF MONOSYMMETRIC I-BEAMS

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RESEARCH REPORT R920
SEPTEMBER 2011

ISSN 1833-2781

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Published by:
School of Civil Engineering
The University of Sydney
Sydney NSW 2006
Australia

This report and other Research Reports published by the School of Civil Engineering are available at http://sydney.edu.au/civil

ABSTRACT

Methods of designing steel monosymmetric I-beams against lateral buckling are not well supported by research. For this paper, the inelastic buckling of monosymmetric steel I-beams under moment gradient was studied and compared with design recommendations.

For welded beams in uniform bending, inelastic buckling is initiated at moments which are often close to those which cause first yield in the compression flange. Once initiated, the inelastic buckling resistance remains constant as the slenderness decreases until strain-hardening occurs. For hot-rolled beams in uniform bending, the inelastic buckling resistance increases almost linearly as the slenderness decreases.

Three regimes are significant in the inelastic buckling resistances of hot-rolled monosymmetric beams under moment gradient, depending on which flange yields first and the end moment ratio.

Simple linear approximations of good accuracy were developed for designing hot-rolled monosymmetric beams in uniform bending, while less accurate but conservative approximations were developed for moment gradient. The use of these approximations was illustrated by a worked example.

KEYWORDS

Beams, bending, buckling, design, inelastic, monosymmetry, steel, yield.

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INTRODUCTION

Methods of designing steel monosymmetric I-beams against lateral buckling are not well supported by research. These methods (AISC 2010a, BSI 2005, SA 1998) are generally based on extrapolations from well researched studies of the effects of non-uniform bending on the elastic and inelastic buckling of doubly symmetric I-beams. However, it has been pointed out (Kitipornchai et al 1986) that design methods of allowing for the effects of non-uniform bending on the elastic buckling of monosymmetric beams are questionable, while there appears to have been no attempt to justify the extension of inelastic buckling research to monosymmetric beams.

The elastic lateral buckling of doubly symmetric I-beams has been studied extensively since the early work of Timoshenko in 1905 on simply supported beams (Timoshenko 1953). The effects of moment distribution, load height, and concentrated and distributed restraints have been analysed, and there are many summaries of approximate formulae for the maximum moment at elastic buckling (Trahair 1993). Exact solutions for the elastic buckling of simply supported beams under uniform bending are given in many design codes (AISC 2010a, BSI 2005, SA 1998), as are approximations for the effects of non-uniform bending.

The effects of moment distribution and load height on the elastic lateral buckling of monosymmetric I-beams has also been studied, and there is a number of summaries of approximate formulae for the maximum moment at elastic buckling (Anderson and Trahair 1972, Kitipornchai and Trahair 1980, Kitipornchai et al 1986, Wang and Kitipornchai 1986, Helwig et al 1997). Accurate solutions may be obtained by using finite element programs (Trahair 1993) such as the user-friendly program PRFELB (Papangelis et al 1993). Exact solutions for the elastic buckling of simply supported beams under uniform bending are used in AS4100 (SA 1998) and approximations in the AISC Commentary (AISC 2010b), as are approximations for the effects of non-uniform bending.

The inelastic lateral buckling of doubly symmetric steel I-beams has also been studied, and early research has been summarised in Trahair (1983). Inelastic buckling is influenced by the residual stresses induced during manufacture, which cause early yielding at the compression flange tips, with consequent reductions in the buckling resistance. The yielding patterns are monosymmetric, and so also are the reduced flexural and torsional stiffnesses of the flanges and web (Trahair and Kitipornchai 1972). The effects of residual stresses on the inelastic buckling of hot-rolled beams are accounted for in the AISC Specification (AISC 2010a), although there appears to be no allowance for the different residual stress patterns in welded beams (Fukumoto and Itoh 1981). There are few studies of the inelastic lateral buckling of monosymmetric I-beams (Nethercot 1973).

This paper investigates the inelastic lateral buckling of monosymmetric I-beams under uniform and non-uniform bending. The geometry of the beams studied is shown in Fig. 1. The material properties given in Fig. 2 include reduced moduli E_s, G_s for the inelastic regions which are based on previous studies of inelastic buckling summarised in Trahair (1983, 1993). The residual stresses f_r assumed for welded and hot-rolled beams are shown in Fig. 3. The flange residual stresses in the larger flange are simplified versions of those used in other inelastic buckling studies (Trahair 1983, 1993). Reduced residual stresses are assumed for the smaller flange. No residual stresses are assumed for the web, because web yielding has comparatively little effect on lateral buckling.

UNIFORM BENDING

METHOD OF SOLUTION

The distribution of the total strain across a welded beam section is shown in Fig. 4a as the sum of the residual strains ε_r and the bending strains defined by the web strains ε_{wl}, ε_{ws} at the larger and smaller flanges. The distribution of the total stress f is shown in Fig. 4b. A method of calculating the inelastic uniform bending buckling moment M_{iu} which corresponds to an assumed value of the web large flange bending strain ε_{wl}, is as follows:

(a) Determine the value of the web small flange bending strain ε_{ws} and the corresponding distribution of the total stress f which satisfy the axial force equilibrium equation

$$N = \int_A f dA = 0 \tag{1}$$

in which A is the area.

(b) Determine the inelastic buckling moment M_{iu} using

$$M_{iu} = -\int_A f y_l dA \tag{2}$$

in which y_l is the distance from the larger flange centroid to the stress point.

(c) Determine the inelastic minor axis flange flexural rigidities EI_{li}, EI_{si} and the inelastic section flexural and torsional rigidities EI_i, GJ_i by summing the contributions from the elastic and yielded regions using the elastic moduli E, G for the elastic regions of the section and the strain-hardening values E_s, G_s for the yielded regions.

(d) Determine the inelastic shear centre distance from the larger flange centroid y_{0i} and the inelastic warping rigidity EC_{wi} using

$$y_{0i} = \frac{b_w EI_{si}}{EI_{li} + EI_{si}} \tag{3}$$

$$EC_{wi} = EI_{li} y_{0i}^2 + EI_{si}(b_w - y_{0i})^2 \tag{4}$$

in which b_w is the distance between flange centroids.

(e) Determine the inelastic monosymmetry section constant β_{xi} using

$$\beta_{xi} = \frac{1}{M_{iu}} \int_A (f - f_r)\{x^2 + (y_l - y_{0i})^2\} dA \tag{5}$$

The inclusion of the residual stress f_r in this equation compensates approximately for the residual stress distribution having a non-zero torsional stress resultant during twisting (Wagner 1936).

(f) Determine the beam length L_{iu} by solving

$$M_{iu} = \sqrt{\frac{\pi^2 EI_{yi}}{L_{iu}^2}} \left\{ \sqrt{\left[GJ_i + \frac{\pi^2 EC_{wi}}{L_{iu}^2} + \left\{ \frac{\beta_{xi}}{2} \sqrt{\frac{\pi^2 EI_{yi}}{L_{iu}^2}} \right\}^2 \right]} + \frac{\beta_{xi}}{2} \sqrt{\frac{\pi^2 EI_{yi}}{L_{iu}^2}} \right\} \tag{6}$$

(g) Determine the elastic uniform bending buckling moment M_{eu} corresponding to this length L_{iu} by substituting L_{iu} for L in

$$M_{eu} = \sqrt{\frac{\pi^2 EI_y}{L^2}} \left\{ \sqrt{\left[GJ + \frac{\pi^2 EC_w}{L^2} + \left\{ \frac{\beta_x}{2} \sqrt{\frac{\pi^2 EI_y}{L^2}} \right\}^2 \right]} + \frac{\beta_x}{2} \sqrt{\frac{\pi^2 EI_y}{L^2}} \right\} \tag{7}$$

in which I_y, J, C_w are the minor axis second moment of area, torsion constant, and warping constant of the section respectively, and β_x is the monosymmetry section constant given by

$$\beta_x = \frac{1}{I_x} \left\{ \int_A y(x^2 + y^2) dA \right\} - 2y_0 \tag{8}$$

in which I_x is the major axis second moment of area and y_0 is the shear centre (S) distance from the centroid (C).

RESULTS FOR WELDED BEAMS

The variations of the dimensionless inelastic buckling moments M_{iu}/M_p (in which M_p is the full plastic moment of the section) with the uniform bending modified slenderness

$$\lambda_u = \sqrt{(M_p / M_{eu})} \tag{9}$$

are shown in Fig. 5 for monosymmetric I-beams with the welded residual stresses shown in Fig. 3a. The values of b_s / b_l for the different curves shown in Fig.5 are positive when the moment causes compression in the larger flange, and negative when the moment causes compression in the smaller flange (this convention is also used in the later Figs 6, 7, 9, and 10).

Also shown in Fig. 5 are the dimensionless elastic buckling moments M_{eu}/M_p, and the dimensionless strain-hardening buckling moments M_{su}/M_p in which M_{su} is obtained by using $E = E_s$ and $G = G_s$ in Equation 7. At high slendernesses, the values of M_{iu}/M_p are close to the elastic values, while at low slendernesses they are close to the strain-hardening values.

Between these slendernesses, the values of M_{iu}/M_p are constant. Similar results were reported by Nethercot (1974). When the larger flange is in compression (b_s/b_l positive), the constant values increase as the beams become more monosymmetric, while the reverse is true when the smaller flange is in compression (b_s/b_l negative). These constant values often correspond to first yield of the outer regions of the compression flange which causes significant reductions in the values of EI_{yi} and EC_{wi}, as well as changes in the shear centre position y_{0i} and consequent changes in the monosymmetry section constant β_{xi}. These first yield values are given by

$$M_{fy} = (f_y + f_{rc}) S_{xc} \tag{10}$$

in which f_{rc} is the residual stress at the compression flange tip and S_{xc} is the elastic section modulus for the compression flange.

Also shown in Fig. 5 are the nominal design moments of AISC (2005a) (for equal flanged I-beams), EC3 (BSI 2005) and AS 4100 (SA 1998). None of these reflect the calculated inelastic buckling moments, except that the AS 4100 and EC3 low slenderness limits (at which $M_{iu} = M_p$) are close to the strain-hardening limit, while the AISC intermediate slenderness limit (at which $M_{iu} = M_{eu}$) corresponds to the value calculated for the equal flanged beam.

RESULTS FOR BEAMS WITH HOT-ROLLED RESIDUAL STRESSES

The variations of the dimensionless inelastic buckling moments M_{iu}/M_p with the modified slenderness λ_u are shown in Fig. 6 for monosymmetric I-beams with the hot-rolled residual stresses shown in Fig. 3b. Also shown in Fig. 6 are the dimensionless elastic buckling moments M_{eu}/M_p, and the dimensionless strain-hardening buckling moments M_{su}/M_p. At intermediate slendernesses, the values of M_{iu}/M_p are close to the elastic values, while at low slendernesses they are close to the strain-hardening values.

Between these slendernesses, the variations of M_{iu}/M_p are approximately linear. At low slendernesses, the inelastic moments approach the strain-hardening values, and are often close to values which correspond to full plasticity of the compression flange. At intermediate slendernesses they approach the elastic values, and are often close to values which correspond to first yield at the compression flange tip. When the larger flange is in compression (b_s/b_l positive), the values increase as the beams become more monosymmetric, while the reverse is true when the smaller flange is in compression (b_s/b_l negative).

It should be noted that in beams whose compression flange is the larger, early yielding at the tips of the smaller tension flange is delayed by the residual compression stresses there, so that yielding first occurs at the flange web junction where it has little effect on the buckling resistance.

Also shown in Fig.6 is a range of design strengths calculated for compact beams that are capable of reaching the full plastic moment by using the AISC Specification (AISC 2010a,b). These strengths vary little by comparison with the significant variations of the inelastic buckling resistances.

MOMENT GRADIENT

METHOD OF SOLUTION

The inelastic lateral buckling resistances of beams under moment gradient are significantly affected by local reductions in the out-of-plane stiffnesses in the high moment regions of the beams where yielding takes place. Because the moment distribution varies, so do the stiffnesses, and the beams become non-uniform. For this paper, a method of accounting for the local stiffness reductions caused by non-uniform yielding has been used which is based on a method of member strength design by inelastic buckling analysis (Trahair and Hancock 2004), which itself is a development of the method of design by elastic buckling analysis permitted or implied in such codes as the AS 4100 (SA 1998) and EC3 (BSI 2005).

In the code method of design by elastic buckling analysis, the results of an elastic buckling analysis are reduced by code slenderness reduction factors (SA 1998, BSI 2005) based on the design strengths of beams in uniform bending. These reduction factors make allowance for the effects of residual stresses and geometrical imperfections. However, this method does not allow for the local effects of non-uniform yielding on the inelastic buckling resistance.

In the method of member strength design by inelastic buckling analysis, an elastic buckling analysis is carried in which the local effects of non-uniform yielding on the out-of-plane stiffnesses are allowed for by using reduced stiffnesses derived from the code design strengths of beams in uniform bending. This method has been tested against code formulations (SA 1998) for the lateral buckling strengths of beams under double curvature bending, beams with central concentrated loads acting away from the shear centre, and beams with elastic end restraints (Trahair and Hancock 2004). It has also been used for columns, beam-columns (Trahair and Hancock 2004), frames (Trahair 2009), and cantilevers (Trahair 2010).

For this paper, this method of design by inelastic buckling analysis used out-of-plane section stiffness reductions determined from the inelastic lateral buckling strengths M_{iu} shown in Fig. 6 for monosymmetric beams with the hot-rolled residual stresses shown in Fig. 3b. These strengths were used to calculate the reduced section stiffness factors $\alpha = M_{iu}/M_{eu}$ whose variations with the dimensionless moments M_{iu}/M_p are shown in Fig. 7. These factors were used to reduce both the elastic moduli E, G. These factors represent the results of integrations over the section of the contributions to the section properties from the elastic and yielded regions described previously in the method of solution for uniform bending.

The beam length L_i corresponding to an assumed inelastic buckling moment M_i was determined iteratively by using the reduced moduli in the elastic buckling program PRFELB (Papangelis et al 1993). Up to 25 nodes were used, with close spacing in the regions where the reduced moduli varied significantly. Up to 13 different elements with averages of the nodal values of the reduced moduli were used.

The program PRFELB was also used with the length L_i to determine the corresponding elastic buckling moment M_e (which includes the effects of both monosymmetry and moment gradient) and the corresponding modified slenderness

$$\lambda = \sqrt{(M_p / M_e)} \tag{11}$$

When this method is applied to monosymmetric beams in uniform bending, it produces predictions of the inelastic buckling moments M_i which are identical to the values of M_{iu} shown in Fig. 6.

RESULTS

The variations of the dimensionless inelastic buckling moments M_i/M_p with the modified slendernesses λ for monosymmetric beams with unequal end moments M, βM are shown in Figs 8-10 for flange width ratios b_s/b_l = 1.0, 0.6, and 0.2.

For the beams with b_s/b_l = 1.0 (double symmetry) shown in Fig. 8, the effects of unequal end moments cause the inelastic buckling moment M_i to increase significantly towards the elastic buckling moment M_e as the end moment ratio β increases from –1 (uniform bending) towards +1 (double curvature bending). These increases are similar to but significantly less than those of the AISC Specification (AISC 2010a), also shown in Fig. 8.

There are 3 different regimes for the dimensionless inelastic buckling moments M_i / M_p of the highly monosymmetric beams with b_s/b_l = 0.2 shown in Fig. 10. The lowest regime is for beams for which the maximum end moment causes compression in the smaller flange (b_s/b_l negative). For this regime, there is a steady increase in the value of M_i/M_p as the end moment ratio β increases from –1 (uniform bending) to +1 (double curvature bending).

Similar increases occur for the highest regime, for which the maximum moment causes compression in the larger flange (b_s/b_l positive) and the end moment ratio β varies between -1 and +0.57. For this regime, first yield in compression is caused by the larger end moment and occurs in the larger flange.

The third regime is for beams whose maximum moment causes compression in the larger flange (b_s/b_l positive) and the end moment ratio β varies between +0.57 and +1. For this regime first yield is caused by the smaller end moment, and occurs in the smaller flange. For this regime, the dimensionless inelastic buckling moments M_i / M_p decrease as the end moment ratio β increases from +0.57 to +1.

For the moderately monosymmetric beams with b_s/b_l = 0.6 shown in Fig. 9, the variations of the dimensionless inelastic buckling moments M_i/M_p with the end moment ratio β are similar to but less pronounced than those shown in Fig. 10 for the highly monosymmetric beams with b_s/b_l = 0.2.

DESIGN METHODS

UNIFORM BENDING

Although it is possible to develop high accuracy approximations for predicting the inelastic buckling moments of monosymmetric I-beams in uniform bending, it is unlikely that these will be included in design codes whose lateral buckling strengths are based on inelastic buckling, such as the AISC Specification (AISC 2010a). Instead, simple linear approximations for the nominal uniform bending maximum moment capacity M_{nu} are presented in this section which are of reasonable accuracy while still being suitable for design. These approximations (for compact beams) are as follows:

$$\begin{aligned}
&\text{for} \quad 0 \le \lambda_u < 0.2, && M_{nu} = M_p \\
&\text{for} \quad 0.2 \le \lambda_u < \lambda_{uy}, && M_{nu} = M_{isu} - (M_{isu} - M_{ieu})\left(\frac{\lambda_u - 0.2}{\lambda_{uy} - 0.2}\right) \\
&\text{for} \quad \lambda_{uy} \le \lambda_u, && M_{nu} = M_{eu}
\end{aligned} \tag{12}$$

In these equations, λ_u is the modified slenderness given by Equation 9,

$$\lambda_{uy} = \sqrt{(M_p / M_{ieu})} \tag{13}$$

is the value of λ_u for which $M_{ieu} = M_{eu}$, M_{eu} is the elastic buckling moment given by Equation 7 (which includes the effects of monosymmetry), and the values of M_{ieu} and M_{isu} may be approximated by

$$\begin{aligned}
&\text{for} \quad 0 < \frac{S_{xc}}{S_{xc} + S_{xt}} \le 0.5, && M_{ieu} = M_{fy} \\
&\text{for} \quad 0.5 \le \frac{S_{xc}}{S_{xc} + S_{xt}} < 1.0, && M_{ieu} = M_{fy} - 3M_p\left(\frac{S_{xc}}{S_{xc} + S_{xt}} - 0.5\right)^2
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
&\text{for} \quad 0 < \frac{S_{xc}}{S_{xc} + S_{xt}} \le 0.5, && M_{isu} = M_{fp} \\
&\text{for} \quad 0.5 \le \frac{S_{xc}}{S_{xc} + S_{xt}} < 1.0, && M_{isu} = \left(0.3 f_y \sqrt{S_{xc} S_{xt}} + 0.7 M_{fp}\right) \quad \text{while} \quad M_{fp} \le M_p \\
& && M_{isu} = \left(0.3 f_y \sqrt{S_{xc} S_{xt}} + 0.7\right) \quad \text{while} \quad M_{fp} \ge M_p
\end{aligned} \tag{15}$$

in which S_{xt} is the elastic section modulus for the tension flange, M_{fy} is the moment at which the compression flange first yields at the flange tip given by Equation 10 (which includes an allowance for the residual stress), and

$$M_{fp} = f_y S_{xc} \tag{16}$$

is the moment at which the compression flange fully yields.

The accuracy of the approximations of Equations 14 and 15 is demonstrated in Fig. 11.

MOMENT GRADIENT

The complexity of the variations of the inelastic buckling moments with the end moment ratio β (demonstrated in Fig. 10) prevents the development of simple approximations of close accuracy which are suitable for design. Instead, the linear approximations developed for uniform bending are extended to produce conservative approximations for the nominal maximum moment capacity M_n. These approximations (for compact beams) are as follows:

$$\begin{aligned}
&\text{for} \quad 0 \le \lambda < 0.2, && M_n = M_p \\
&\text{for} \quad 0.2 \le \lambda < \lambda_y, && M_n = M_{is} - (M_{is} - M_{ie})\left(\frac{\lambda - 0.2}{\lambda_y - 0.2}\right) \le M_p \\
&\text{for} \quad \lambda_y \le \lambda, && M_n = M_e
\end{aligned} \tag{17}$$

In these equations, λ is the modified slenderness given by Equation 11,

$$\lambda_y = \sqrt{(M_p / M_{ie})} \tag{18}$$

is the value of λ for which $M_{ie} = M_e$, and M_e is the elastic buckling moment (which includes the effects of moment gradient and monosymmetry). Values of M_e may be obtained from available numerical solutions (Kitipornchai et al 1986) or computer programs such as PRFELB (Papangelis et al 1993). The values of M_{ie} and M_{is} may be approximated as follows:

When the larger end moment M causes compression in the smaller flange,

$$\begin{aligned} M_{ie} &= M_{fys} \\ M_{is} &= M_{fps}(1.26 + 0.18\beta - 0.08\beta^2) \end{aligned} \tag{19}$$

in which

$$\begin{aligned} M_{fys} &= (f_y + f_{rs})S_{xs} \\ M_{fps} &= f_y S_{xs} \end{aligned} \tag{20}$$

in which f_{rs} is the (negative) residual stress at the tip of the smaller flange.

When the larger end moment M causes compression in the larger flange, and $-1 \le \beta \le M_{fys} / M_{fyl}$, in which

$$M_{fyl} = (f_y + f_{rl})S_{xl} \tag{21}$$

in which f_{rl} is the (negative) residual stress at the tip of the larger flange, then

$$\begin{aligned} M_{ie} &= M_{fyl} \\ M_{is} &= M_{isul}(1.34 + 0.278\beta - 0.07\beta^2) \end{aligned} \tag{22}$$

in which M_{isul} is the value of M_{isu} for the larger flange obtained from Equation 15.

When the larger end moment M causes compression in the larger flange, and $M_{fys} / M_{fyl} \le \beta \le 1$,

$$\begin{aligned} M_{ie} &= M_{fys} / \beta \\ M_{is} &= M_{fps}(3.4 - 2\beta) \end{aligned} \tag{23}$$

WORKED EXAMPLE

EXAMPLE

Determine the nominal maximum design moment for a beam having the geometry, properties and residual stresses shown in figs 1-4 when $b_s/b_l = t_s/t_l = 0.6$, $L = 4000$ mm, $\beta = 0.8$, and the maximum moment causes compression in the larger flange. For this beam, PRFELB predicts that the maximum moment at elastic buckling is $M_e = 1.627$ E8 Nmm, while the full plastic moment is $M_p = 1.271$ E8 Nmm and the elastic section moduli are $S_{xl} = 5.338$ E5 mm^3 and $S_{xs} = 3.061$ E5 mm^3.

SOLUTION

Using (11), $\lambda = \sqrt{(1.271/1.627)} = 0.8839 > 0.2$

Using (20), $M_{fys} = 300 \times (1 - 0.3 \times 0.6) \times 3.061\text{E}5 = 0.7530\,\text{E}8$ Nmm

Using (21), $M_{fyl} = 300 \times (1 - 0.3) \times 5.338\,\text{E}5 = 1.121\,\text{E}8$ Nmm

Thus, $M_{fys} / M_{fyl} = 0.7530\,\text{E}8 / 1.121\,\text{E}8 = 0.6717 < 0.8 = \beta$, then

Using (20), $M_{fps} = 300 \times 3.061\text{E}5 = 0.9183\,\text{E}8$ Nmm

Using (23), $M_{ie} = 0.7530\,\text{E}8 / 0.8 = 0.9413\,E8$ Nmm

$M_{is} = 0.9183\,\text{E}8 \times (3.4 - 2 \times 0.8) = 1.653\,\text{E}8$ Nmm

Using (18) $\lambda_y = \sqrt{(1.271\,\text{E}8 / 0.9413\,\text{E}8)} = 1.162 > 0.8839 = \lambda$

Using (17), $M_n = \left\{ 1.653 - (1.653 - 0.9413) \times \dfrac{(0.8839 - 0.2)}{(1.162 - 0.2} \right\} \text{E}8 = 1.147\,\text{E}8\ \text{Nmm} < 1.271\text{E}8 = M_p$

CONCLUSIONS

For this paper, the inelastic buckling of monosymmetric steel I-beams under moment gradient was studied and compared with design recommendations. Variations in the bending moment along a beam cause variations in the effective moduli and in a beam's resistance to inelastic lateral buckling. These variations were accounted for by adapting a method of design by inelastic buckling analysis in which reduced moduli were used in elastic buckling analyses.

Beams with residual stress distributions typical of welded and hot rolled beams were assumed. For welded beams in uniform bending, inelastic buckling is initiated at moments which are often close to those which cause first yield in the compression flange. Once initiated, the inelastic buckling resistance remains constant as the slenderness decreases until strain-hardening occurs. Design code recommendations do not reflect this behaviour.

For hot-rolled beams in uniform bending, the inelastic buckling resistance increases almost linearly as the slenderness decreases, until strain-hardening occurs. The resistance also increases significantly as the relative size of the compression and tension flanges increases, but this is not reflected in the AS (SA 1998) and EC3 (BSI 2005) design codes. The design rules of the AISC (2010a) display these trends, but with significant differences.

Three regimes are significant in the inelastic buckling resistances of hot-rolled monosymmetric beams under moment gradient. For beams for which the maximum moment causes compression in the smaller flange, the resistance is low and increases with moment gradient. For beams for which the maximum moment causes yielding in the larger flange before the minimum end moment causes yielding in the smaller flange, the resistance is high and increases with moment gradient. For beams for which the minimum end moment causes yielding in the smaller flange before the maximum moment causes yielding in the larger flange, the resistance is moderate, and decreases with moment gradient.

Simple linear approximations of good accuracy were developed for designing hot-rolled monosymmetric beams in uniform bending, while less accurate but conservative approximations were developed for moment gradient. These approximations require the use of predictions of the maximum moment at elastic buckling. Closed form solutions for these are available for uniform bending, but available numerical solutions (Kitipornchai et al 1986) or computer programs such as the user-friendly PRFELB (Papangelis et al 1993) must be used for moment gradient. The use of these approximations was illustrated by a worked example.

APPENDIX 1 - REFERENCES

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APPENDIX 2 – NOTATION

SUBSCRIPTS

c, t	Compression or tension
e, i, s	Elastic, inelastic, or strain-hardening buckling
p, y	Flange at full plasticity or first yield
l, s	Larger or smaller flange
r	Residual
u	Uniform bending
x, y	Principal axis values

PRINCIPAL NOTATION

A	Area of cross section
b	Flange width
b_w	Distance between flange centroids
C_w	Warping section constant
E	Young's modulus of elasticity
f	Total stress
f_y	Yield stress
G	Shear modulus of elasticity
I	Second moment of area
J	Uniform torsion constant
L	Span length
M	Maximum end moment
M_{fp}	Moment which causes full yielding of flange
M_{fy}	Moment which causes first yield of flange
M_n	Nominal design moment capacity
N	Axial force resultant of stresses
S	Elastic section modulus
t	Thickness
t_w	Web thickness
x, y	Principal axis coordinates
y_0	Shear centre coordinate
y_{0l}	Shear centre distance from larger flange centroid
y_l	Distance from larger flange centroid

α	Ratio of inelastic and elastic moduli
β	End moment ratio
β_x	Monosymmetry section constant
ε	Strain
ε_{wl}, ε_{ws}	Web strains at larger and smaller flanges
λ	Modified slenderness
λ_y	Modified slenderness for $M_i = M_e$

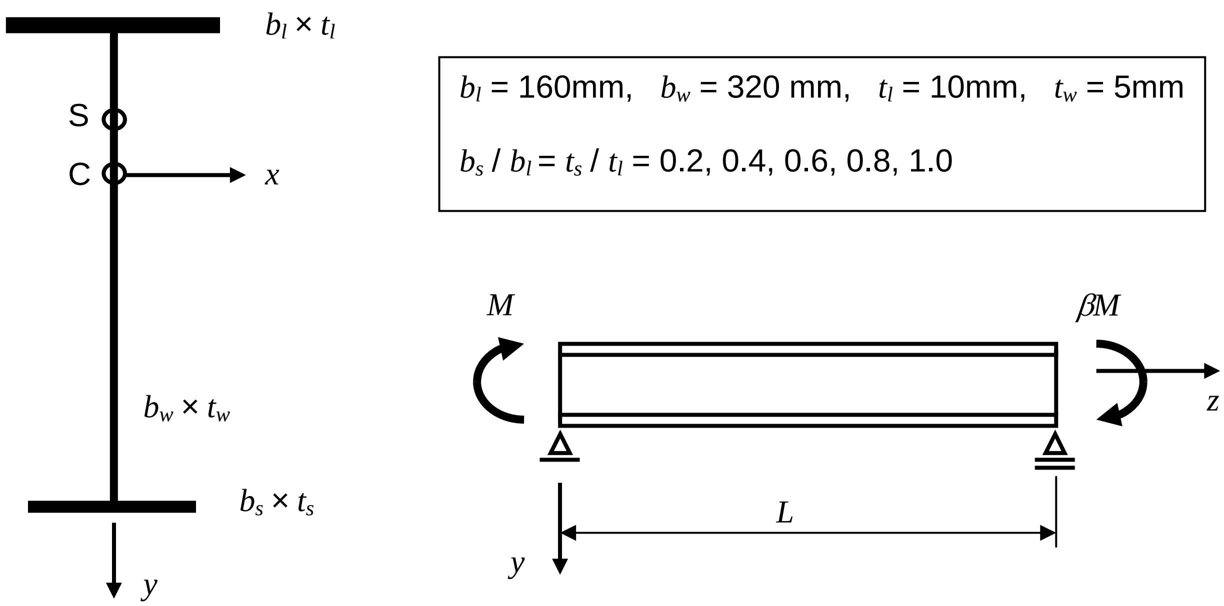


Fig. 1 Geometry and Loading of Monosymmetric Beams

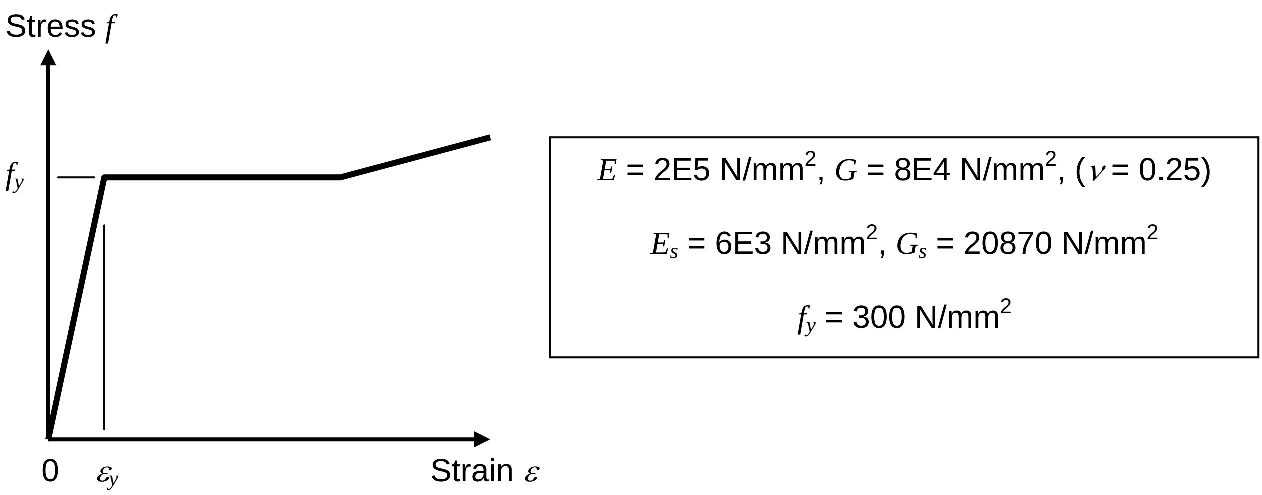


Fig. 2 Material Properties of Monosymmetric Beams

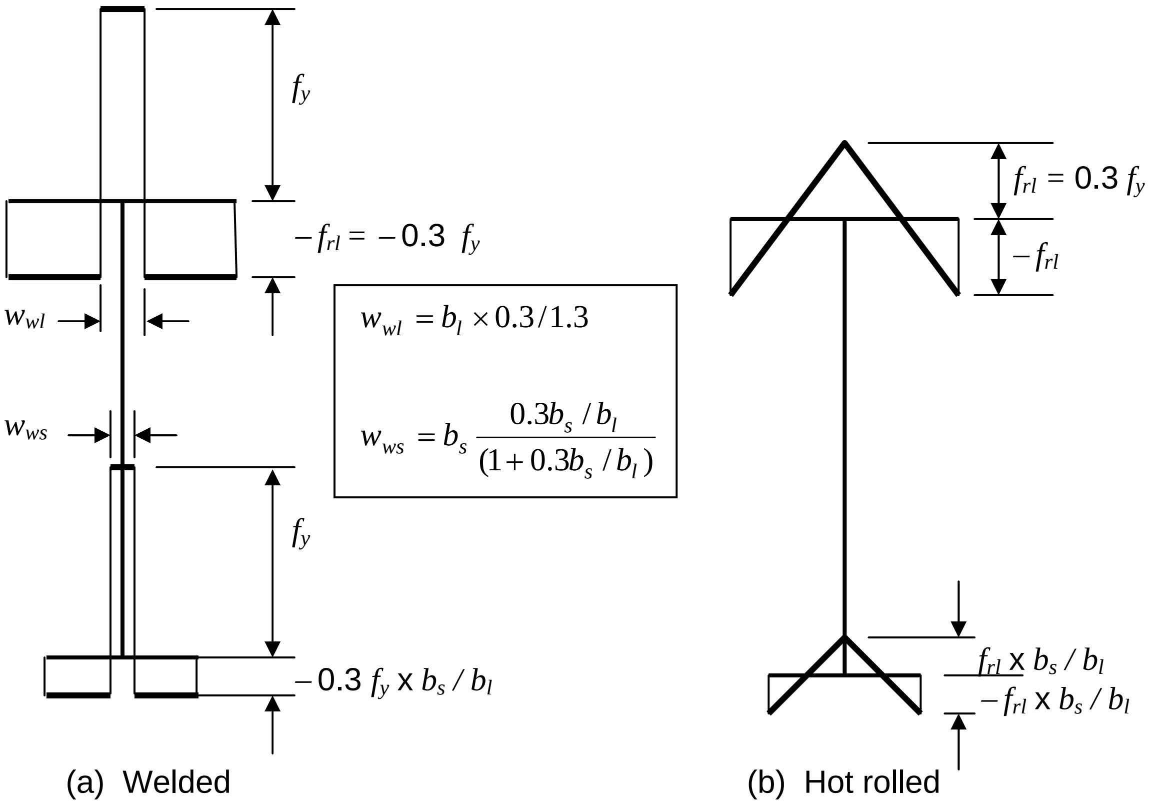
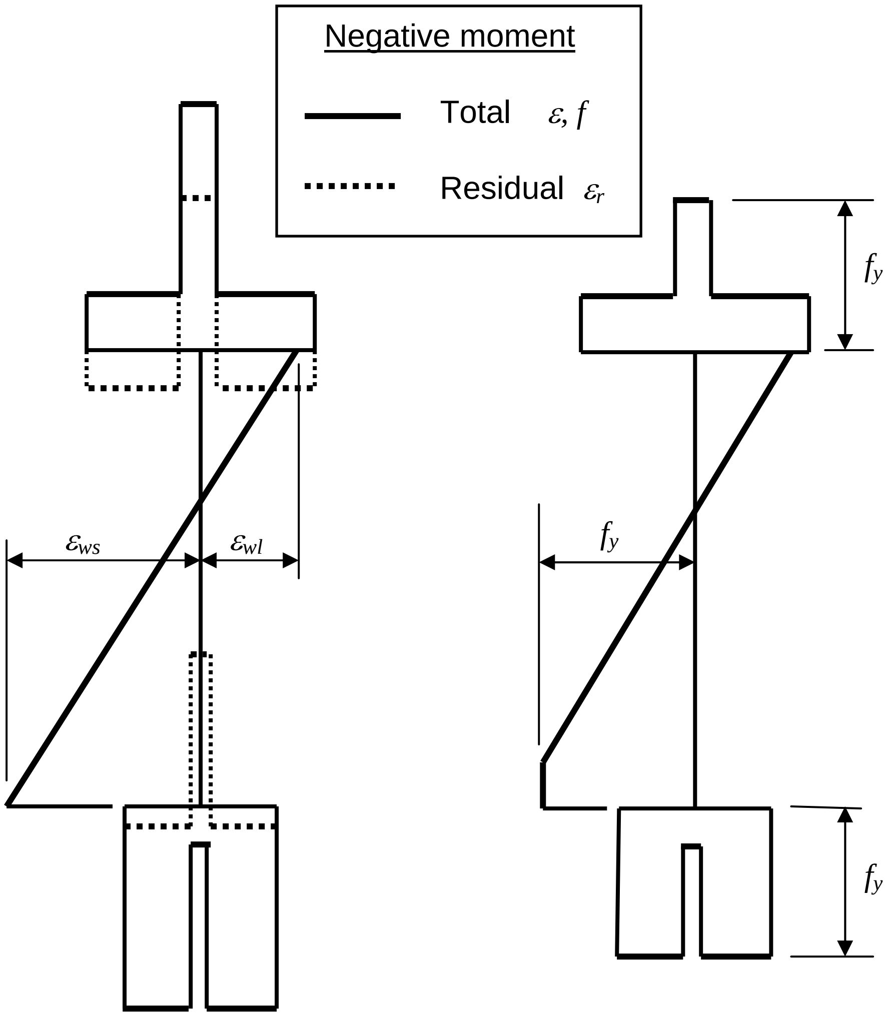


Fig. 3 Residual Stresses in Monosymmetric Beams



(a) Strains ε_r, ε (b) Stresses f

Fig. 4 Total Strains and Stresses in Welded Beams

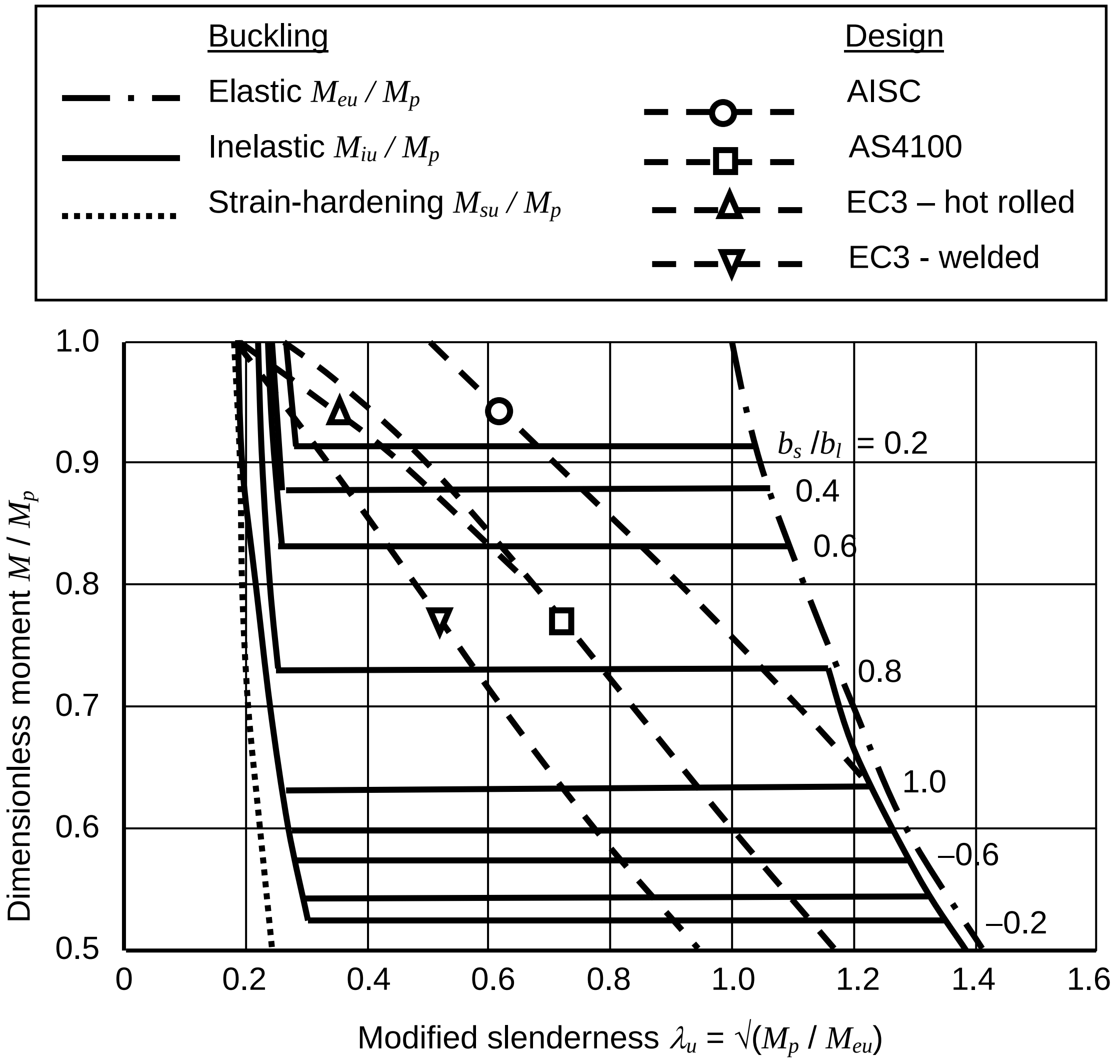


Fig. 5 Inelastic Buckling of Welded Beams – Uniform Bending

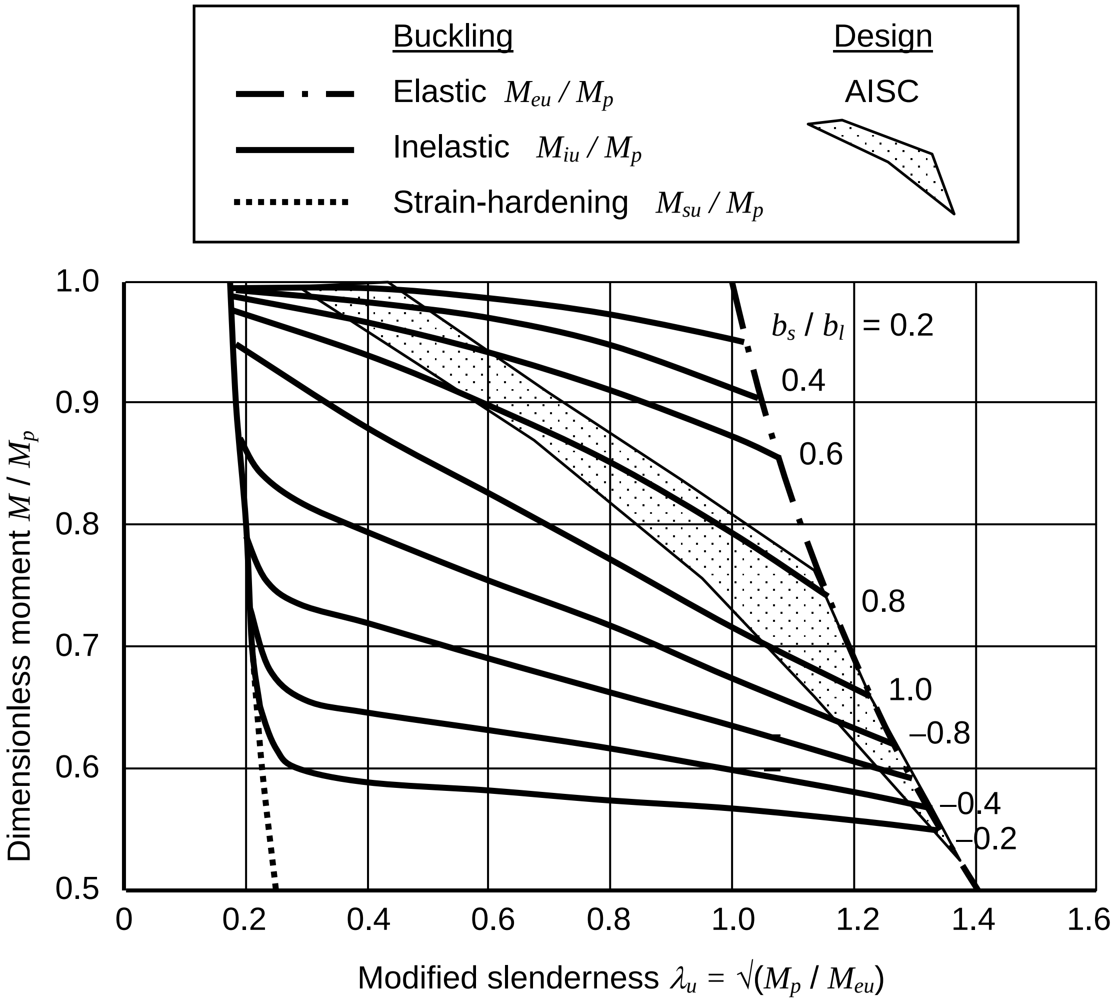


Fig. 6 Inelastic Buckling of Hot Rolled Beams – Uniform Bending

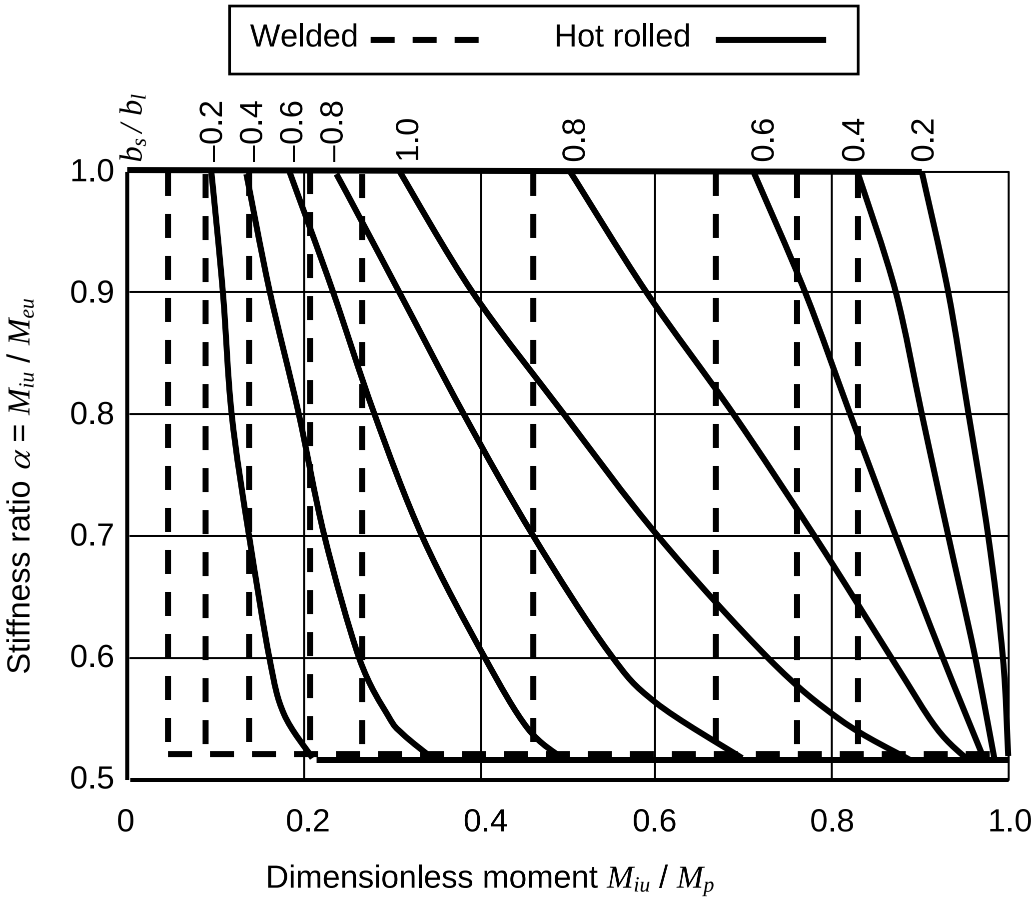


Fig. 7 Inelastic Stiffnesses of Monosymmetric Beams

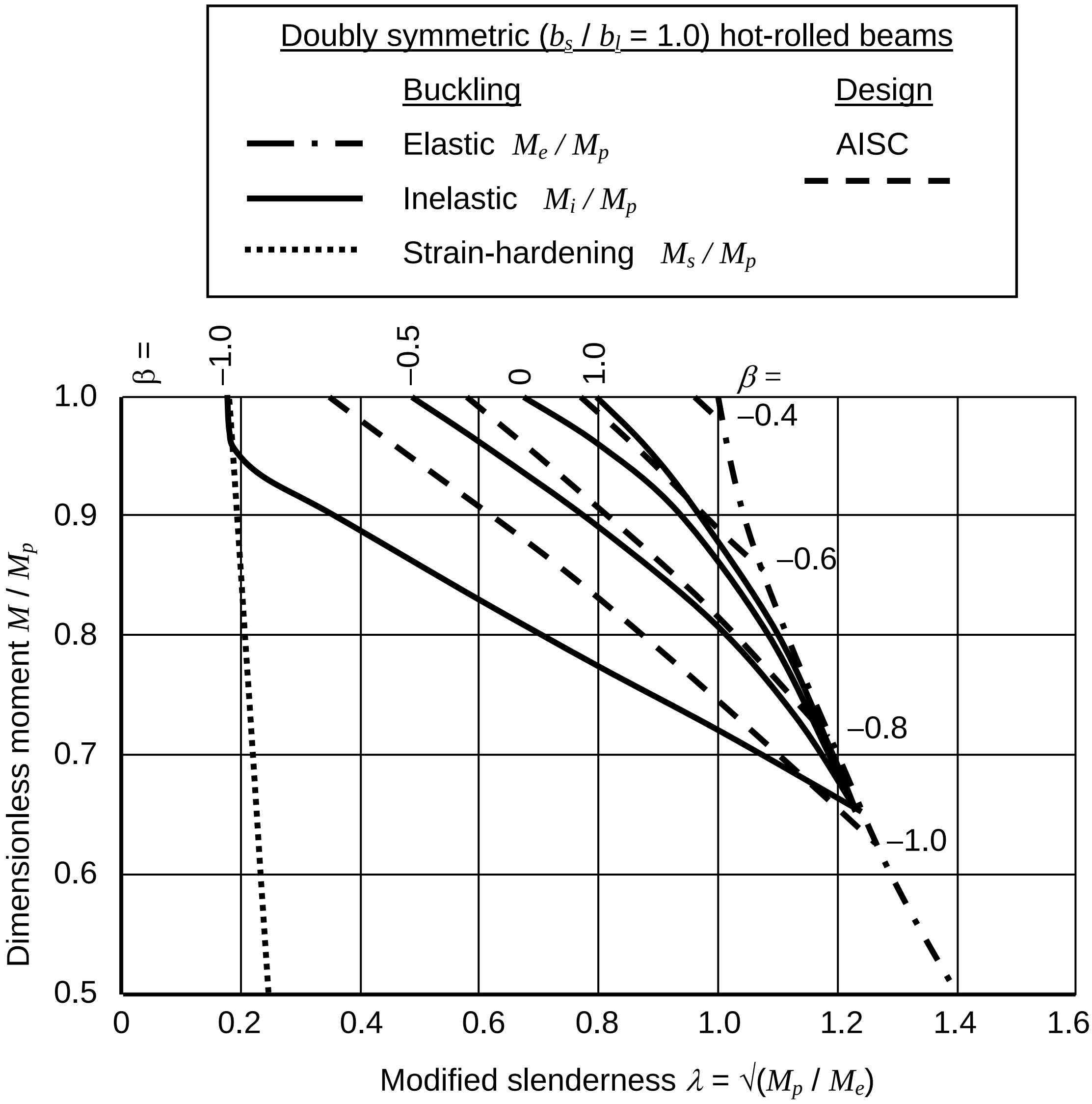


Fig. 8 Inelastic Buckling Under Moment Gradient

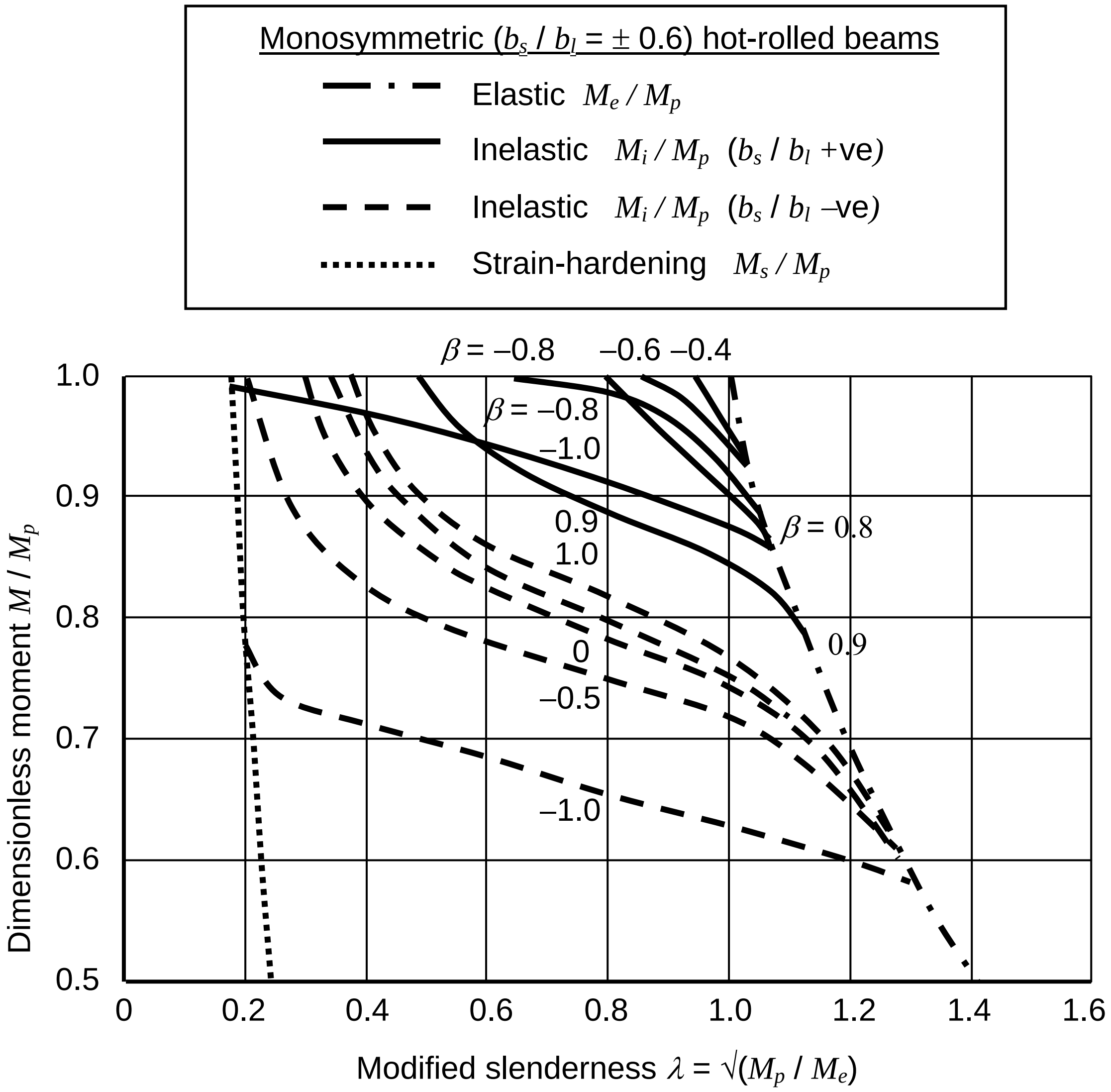


Fig. 9 Inelastic Buckling Under Moment Gradient

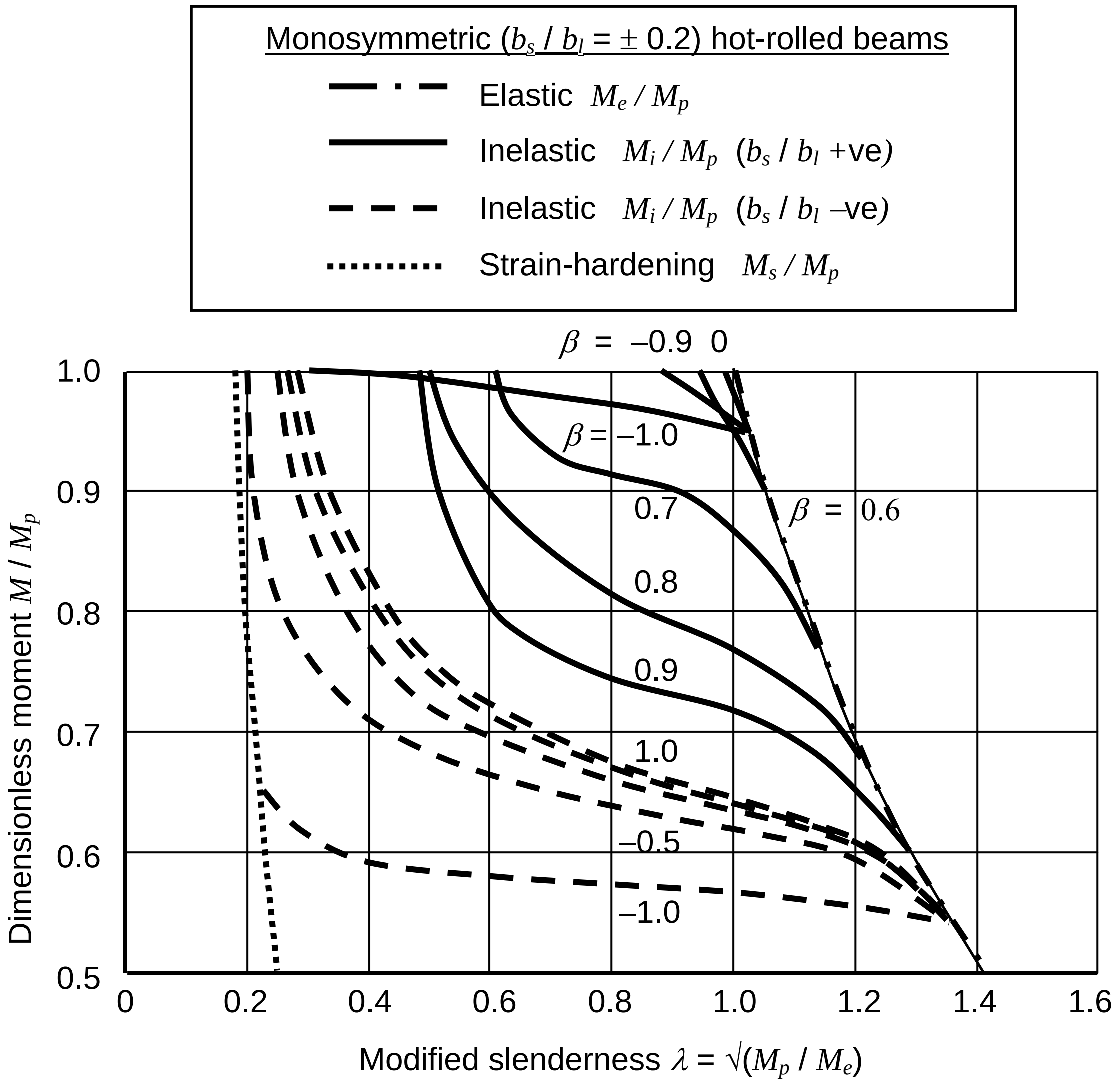


Fig. 10 Inelastic Buckling Under Moment Gradient

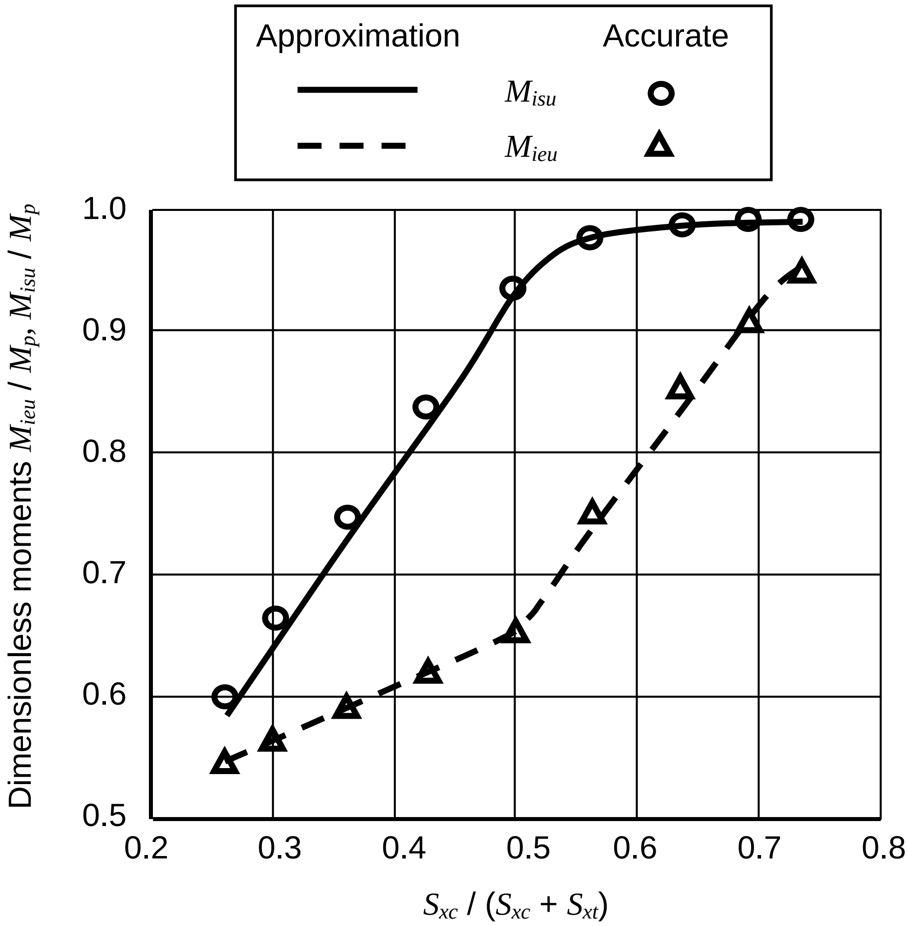


Fig. 11 Compression Flange “Yield” and “Plastic” Moments