SHEAR EFFECT ON CRUCIFORM POST-BUCKLING

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ABSTRACT

The elastic post-torsional-buckling behaviour of a simply supported cruciform column has been analysed, and its strength has been approximated by assuming that it fails when it first yields due to the maximum normal stress induced by the axial compression.

However, torsional shear stresses are induced in the post-buckling regime. The purpose of this note is to investigate the effects of these shear stresses on the first yield strength.

KEYWORDS

Buckling, Columns, Cruciforms, Normal stress, Post-buckling, Shear stress, Steel, Torsion, Yield

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1. INTRODUCTION

A recent paper [1] on simply supported cruciform steel columns (Fig. 1) investigated the effects on strength of elastic local and torsional buckling and post-buckling, and of initial twist and residual stresses. However, the analysis of the post-torsional-buckling behaviour, was approximated by ignoring shear stresses and assuming that failure occurs at first yield due to the maximum normal stress induced by the axial compression.

The purpose of this note is to investigate the effect on the first yield strength of the torsional shear stresses induced in the post-buckling regime.

2. ANALYSIS OF ELASTIC POST-BUCKLING

Elastic torsional buckling of a simply supported cruciform column (with leg widths b and thicknesses t as shown in Fig. 1b) occurs when the axial compression N reaches the elastic buckling value [2-4]

$$N_{oz} = (GJ + \pi^2 EI_w / L^2)/r_0^2 \tag{1}$$

in which GJ is the uniform torsional rigidity, EI_w is the warping rigidity and

$$r_0^2 = (I_x + I_y)/A = b^2/3 \tag{2}$$

in which I_x, I_y are the principal axis second moments of area and A is the area of the cross-section. For cruciform sections, the torsion section constant

$$J = 4bt^3/3 \tag{3}$$

is small, while the warping section [4]

$$I_w = b^3 t^3 / 9 \tag{4}$$

is very small and often neglected. Thus the torsional buckling load is usually taken as

$$N_{oz} = GJ / r_0^2 \tag{5}$$

After torsional buckling, the cruciform undergoes twist rotations (Fig. 1b) which may be approximated by

$$\phi = \phi_m \sin \pi z / L \tag{6}$$

It is assumed that the axial compression N acts through rigid end platens so that the end displacements w are constant, as shown in Fig. 1a. These displacements are combinations of those due to elastic axial straining and to axial shortening caused by the twist rotations.

The shortening displacements are

$$w_s = \frac{1}{2}\int_0^L \left(\frac{dv}{dz}\right)^2 dz \tag{7}$$

in which

$$v = x\phi \tag{8}$$

whence

$$w_s = \frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{x^2}{2} \tag{9}$$

The displacements due to axial straining are

$$w_f = w - w_s \tag{10}$$

so that the elastic compression stresses are

$$f = Ew_f / L = Ew/L - \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{L}{2}\frac{x^2}{2} \tag{11}$$

which may be written as

$$f = Ew/L - N_\phi \frac{3}{5bt}\left(\frac{x}{b}\right)^2 \tag{12}$$

in which

$$N_\phi = \frac{E}{L}\frac{\pi^2}{L^2}\phi_m^2 \frac{4Lb^3 t}{15} \tag{13}$$

The axial compression force is

$$N = \int_A f dA = EAw / L - 4N_\phi / 5 \tag{14}$$

so that

$$f = \frac{N}{A} + \frac{5}{4}\frac{N_\phi}{A}\left(1 - \frac{3x^2}{b^2}\right) \tag{15}$$

and the maximum compression stress is

$$f_m = \frac{N}{A} + \frac{5}{4}\frac{N_\phi}{A} \tag{16}$$

The stresses f cause elastic torsional buckling when the disturbing effect of these stresses is equal to the torsional resistance [2-4], so that

$$GJ = \int_A f(x^2 + y^2) dA \tag{17}$$

The corresponding axial compression $N=N_{pz}$ may be obtained by substituting Equation (15) into Equation (17) and integrating, whence

$$GJ = N_{pz}\, r_0^2 - N_\phi r_0^2 \tag{18}$$

and

$$N_{pz} = N_{oz} + N_\phi \tag{19}$$

The post-buckling twist rotations ϕ induce shear stresses τ, the maximum value of which may be approximated [5] by

$$\tau_m = \frac{GJ\phi_m' t}{J} = G\phi_m \pi t / L \tag{20}$$

in which $' \equiv d / dz$. This equation may be expressed as

$$\tau_m = \sqrt{\frac{15G}{E}\frac{N_\phi}{A}\frac{N_{oz}}{A}} \tag{21}$$

3. ANALYSIS OF FIRST YIELD

The maximum normal stress f_m due to the axial compression and the maximum torsional shear stress τ_m may be combined as an equivalent von Mises stress

$$f_{em} = \sqrt{f_m^2 + 3\tau_m^2} \tag{22}$$

First yield at $N = N_{fy}$ occurs when

$$f_{em} = f_y \tag{23}$$

so that

$$N_y^2 = \left(N_{fy} + \frac{5}{4} N_\phi \right)^2 + \frac{45G}{E} N_\phi N_{oz} \tag{24}$$

4. POST-BUCKLING STRENGTH

It is now assumed that the post-buckling strength of a cruciform column is given by

$$N_{sz} = N_{pz} = N_{fy} \tag{25}$$

at which the column buckles in the post-buckling regime at a load which causes first yield.

Substituting Equations 25 into Equations 19 and 24 leads to

$$A_1 (N_{sz} / N_y)^2 + A_2 (N_{sz} / N_y) + A_3 = 0 \tag{26}$$

in which

$$\begin{aligned} A_1 &= 81/16 \\ A_2 &= -\{45/8 - 45G/E\} N_{oz} / N_y \\ A_3 &= \{(25/16 - 45G/E)\}(N_{oz} / N_y)^2 - 1 \end{aligned} \tag{27}$$

which may be solved for the dimensionless strength N_{sz} / N_y.

5. DISCUSSION

The strength assumption of $N_{sz} = N_{pz} = N_{fy}$ can be thought of as corresponding to the situation in which the torsional resistance GJ is exhausted by the buckling effect of the redistributed axial stresses (Equation 17). This is illustrated in Fig. 2, which shows the elastic buckling load N_{pz} increasing and the first yield load N_{fy} decreasing as the maximum twist rotation ϕ_m and the axial stress redistribution increase. Any subsequent increase in ϕ_m after the strength is reached at $N_{sz} = N_{pz} = N_{fy}$ will cause yielding to spread and the resulting inelastic redistribution of axial stress will decrease the post-buckling resistance, so that the strength will decrease.

The variations of the solutions N_{sz} / N_y of Equations 26 and 27 with the torsional slenderness

$$\lambda_{oz} = \sqrt{(N_y / N_{oz})} \tag{29}$$

are shown in Fig. 3. Also shown in Fig. 3 are the solutions

$$\frac{N_{szn}}{N_y} = \frac{5}{9}\frac{N_{oz}}{N_y} + \frac{4}{9} \tag{30}$$

obtained [1] by ignoring the effects of shear stresses. These latter solutions can also be obtained by solving the non-linear torsion equations given in [6].

It can be seen from Fig. 3 that the dimensionless strength N_{sz} / N_y decreases from 1 as λ_{oz} increases from 1, but at a slower rate than the dimensionless elastic buckling load N_{oz} / N_y. The effect of the shear stresses is to reduce the difference between N_{sz} / N_y and N_{oz} / N_y.

It should be noted that while the normal stress distributions are constant along the column length, those of the shear stresses are maximum at the ends but decrease to zero at the column mid length. Thus the effects of the shear stresses on yielding will be restricted to the column ends, while those of the normal stresses will occur at all points. It seems likely, therefore, that the analysis of this note may be somewhat pessimistic with respect to the decreases in the post-buckling strength attributed to the shear stresses.

6. CONCLUSIONS

The presence of shear stresses at the ends of a simply supported cruciform column induced in the post-buckling regime reduces its first yield strength. This reduction has been analysed in this note. However, the reduction may not be as great as predicted, because high shear stresses are confined to the column ends, whereas the normal stresses are constant along the length of the column.

7. REFERENCES

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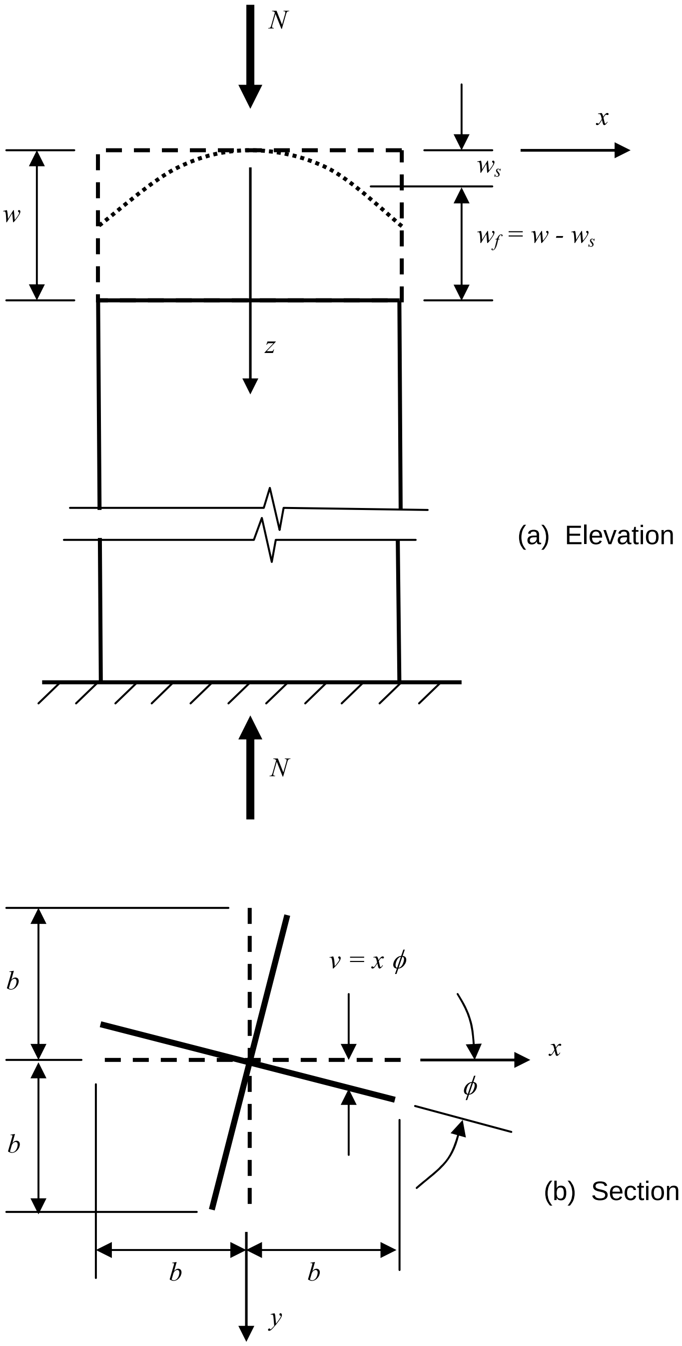
8. NOTATION

A	Area of cross section
b	Leg width
E	Young's modulus of elasticity
f	Normal stress
f_e	Equivalent von Mises stress
f_m	Maximum normal stress
f_y	Yield stress
G	Shear modulus of elasticity
I_x, I_y	Second moments of area about x, y axes
I_w	Warping section constant
J	Uniform torsion section constant
L	Column length
N	Axial compression
N_{fy}	First yield load
N_{oz}	Torsional buckling load
N_{pz}	Torsional post- buckling load
N_{sz}	Torsional post- buckling strength
N_y	Squash load
r_0	Polar radius of gyration
t	Leg thickness
v, w	Displacements in y, z directions
w_f	Displacement due to axial straining
w_s	Axial shortening
x, y	Principal axis coordinates
z	Distance along column
ϕ	Twist rotation
ϕ_m	Maximum twist rotation
λ_{oz}	Modified slenderness for torsional buckling
ν	Poisson's ratio
τ	Shear stress
τ_m	Maximum shear stress



(a) Elevation

(b) Section

Fig. 1 Torsional Post-Buckling of a Cruciform

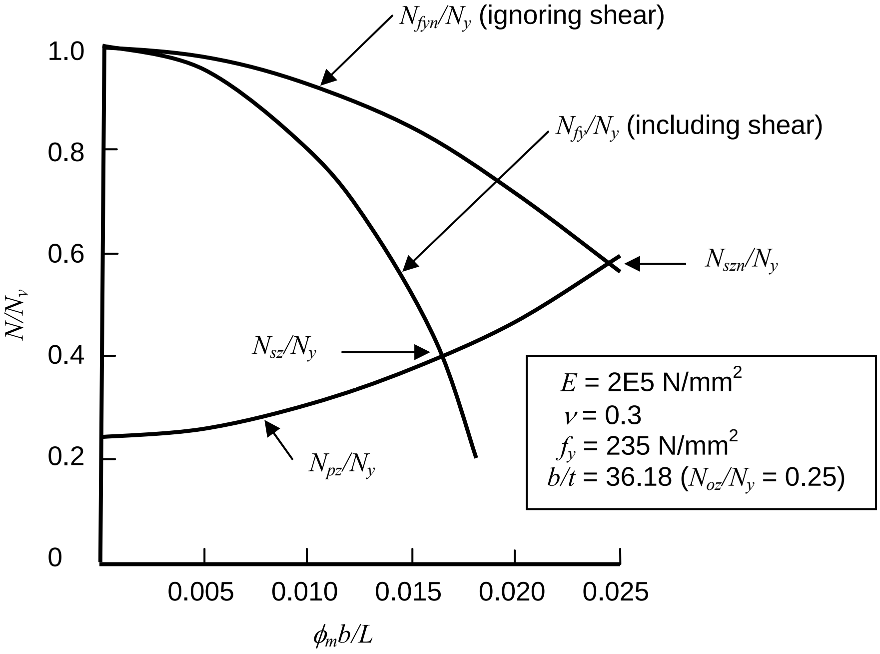


Fig. 2 Post-Buckling and Yielding

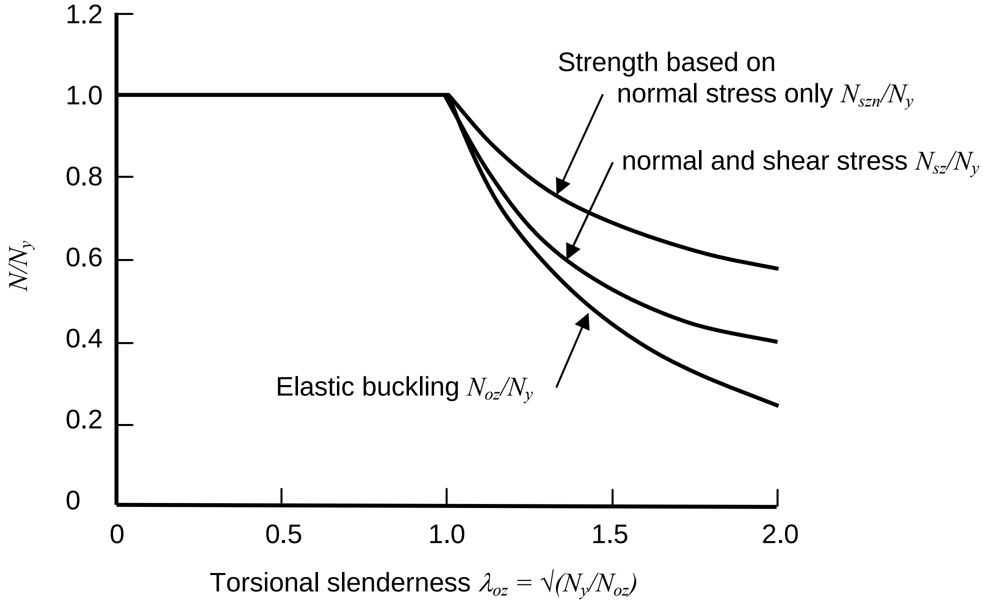


Fig. 3 Effect of Shear Stress on Strength